UNIVERSITE DE GENEVE

CENTRE UNIVERSITAIRE
D'INFORMATIQUE
COMPUTER VISION AND
MULTIMEDIA LABORATORY

Date: 29 June 2004
N° 04.03

TECHNICAL REPORT

Brain-Computer Interface Model: Upper-Capacity Bound, Signal-to-Noise Ratio Estimation, and Optimal Number of Symbols

J. Kronegg, S.Voloshynovskiy, and T. Pun

Computer Vision Group
Computing Science Center, University of Geneva
24 rue du Général Dufour, CH - 1211 Geneva 4, Switzerland

e-mail: Julien.Kronegg@cui.unige.ch

Submited to EURASIP Journal on Applied Signal Processing (JASP), Special issue on Brain-Computer Interface, 2005.

BRAIN-COMPUTER INTERFACE MODEL: UPPER-CAPACITY BOUND, SIGNAL-TO-NOISE RATIO ESTIMATION, AND OPTIMAL NUMBER OF SYMBOLS

Julien Kronegg, Svyatoslav Voloshynovskiy and Thierry Pun

Computer Vision and Multimedia Laboratory, University of Geneva
24 rue General-Dufour, 1211 Geneva 4, Switzerland
Phone: +41 22 3797628, fax: +41 22 3797780, email: Julien.Kronegg@cui.unige.ch
http://vision.unige.ch/MMI/bci

ABSTRACT

The upper capacity bound of a brain-computer interface (BCI) is determined using a model based on Shannon channel theory. This capacity is compared with all bit-rate definitions used in the BCI community (Nykopp, Farwell and Donchin, Wolpaw *et al*); assumptions underlying those definitions and their limitations are discussed. Capacity estimates using Wolpaw and Nykopp bit-rates are computed for various published BCIs. It appears that only Nykopp's bit-rate is coherent with channel theory; Wolpaw's definition leads to an underestimation of the Nykopp's bit-rate and thus should not be used. The usage of a proper bit-rate assessment is motivated and advocated. We also propose an estimation of the typical BCI signal-to-noise ratio, and compute a theoretical optimal number of symbols that is consistent with findings from other researchers.

Keywords : brain-computer interface, BCI, upper capacity bound, bit-rate, information theory, channel theory.

1. INTRODUCTION

In the brain-computer interface (BCI) paradigm, the user think of a specific notion or mental state (e.g. mental calculation, imagination of movement, mental rotation of objects). EEG data collected from the user are then classified and the mental state identified by the machine. This information can be used to drive a specific application (e.g. virtual keyboard [9], [12], cursor control [38], robot control [33]).

During the first 10 years of BCI research (1988-1998), most of the work was focused on discovering new features and classification methods for recognizing mental states, without concentrating too deeply on quantifiable performance comparisons. Results were presented in terms of hit-rates (for cursor control, corresponding to the number of target hit by time unit) or in terms of character-rates (for keyboard applications). It has been further shown that three commonly used BCI design parameters, namely classification speed, number of classes and classifier accuracy are inter-related [12], [18]. Hit-rates or character-rates are therefore not objective measures, since they do not take these three parameters into account.

The first objective BCI performance measure is due to Wolpaw *et al* in 1998 [38], where the bit-rate was defined on the basis of Shannon channel theory [36] with some simplifying assumptions. Bit-rates commonly reported range from 5 to about 25 bits/minute [39]. For the sake of comparison, it is worth note that a keyboard user can achieve 450-1400 bits/minute (assuming error free typing, 32 possible characters and a typing speed of 90-270 characters/minute, depending on user skill).

In this article, we model the BCI as a noisy channel with assumptions that allow to compute the upper capacity bound in term of the amount of reliable information that a BCI user can emit. The present article extends [19] regarding the following issues: various BCIs are compared using both Wolpaw's and Nykopp's bit-rates, the typical signal-to-noise ratio of current BCIs is estimated, and the optimal number of symbols is determined. The article is organized as follows : in Section 2, a review of the noisy channel theory is presented, as a support to the BCI model described in Section 3. Section 4 presents existing bit-rates definitions used in the BCI domain. Discussion of these definitions, and conclusions are presented respectively in Sections 5 and 6.

2. NOISY CHANNEL THEORY

A channel is a communication medium that allows the transmission of information from a sender A to a receiver B. Due to imperfections in that medium, the transmission process is subject to noise and B might receive information differing from the one emitted by A. The simplest noisy channel is the additive noise channel where the received signal Y is the sum of an emitted signal X and some independent noise Z here assumed Gaussian (worst case among all noises with the same variance) (see Figure 1).

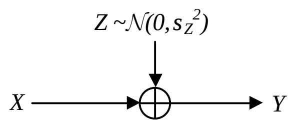

Figure 1: Model of an additive white Gaussian noise channel. X and Y denote the emitted and received symbols, Z the noise.

Since we deal with real, physical input signals, the input signal energy is limited (which also implies that X has zero mean in order to minimize its energy) :

$$E[X^2] \leq \sigma_X^2$$

The information channel capacity is the quantity of reliable information per channel usage:

$$C = \max_{p(x)\,:\,E\left[X^2\right] \le \sigma_X^2} I\left(X;Y\right)$$

This capacity is given in bits per symbol and depends on the mutual information between the input signal X and the output signal Y:

$$I(X;Y) = h(Y) - h(Y|X)$$

$h()$ is the differential entropy (entropy of a continuous random variable) and is defined by (S being the support set of the random variable):

$$h\left(X\right) = \int_S p\left(x\right)\cdot \log p\left(x\right)\cdot dx$$

The channel capacity depends on the input signal distribution as well as on the signal-to-noise ratio (SNR) [7]. We consider below three situations with decreasing capacities : continuous input signal, discrete input signal with equiprobable symbols, discrete input signal with non-equiprobable symbols.

2.1 Continuous input signal

A continuous input signal provides a maximal, continuous capacity. Since $Y=X+Z$ and since the value of X is known:

$$h(Y|X) = h(X=x+Z|X=x) = h(\mathcal{N}(X=x,\ \sigma_Z^2)) = h(Z|X=x)$$

Since X and Z are independents, we have $h(Z|X)=h(Z)$, thus:

$$I\left(X,Y\right) = h\left(Y\right) - h\left(Z\right)$$

The differential entropy of a random variable $\mathcal{N}(\mu,\ \sigma^2)$ does not depends on the mean μ:

$$h\left(\mathcal{N}\left(\mu,\sigma^2\right)\right) = \frac{1}{2}\log_2\left(2\cdot\pi\cdot e\cdot\sigma^2\right) \quad \text{bits}$$

The differential entropy of Y depends on its variance $\mathrm{Var}[Y]=\sigma_X^2+\sigma_Z^2$, thus:

$$h\left(Y\right) \le \frac{1}{2}\log_2\left(2\cdot\pi\cdot e\cdot\left(\sigma_X^2+\sigma_Z^2\right)\right) \quad \text{bits}$$

where the equality holds only for a Gaussian distributed input signal X. We also have the differential entropy of Z:

$$h\left(Z\right) = \frac{1}{2}\log_2\left(2\cdot\pi\cdot e\cdot\sigma_Z^2\right) \quad \text{bits}$$

This leads to the well known continuous input signal capacity (shown in Figure 3):

$$C = \frac{1}{2}\log_2\left(1+\frac{\sigma_X^2}{\sigma_Z^2}\right) \text{bits}$$

or equivalently as a function of the $\mathrm{SNR}=10\cdot\log_{10}(\sigma_X^2/\sigma_Z^2)$:

$$C = \frac{1}{2}\log_2\left(1+10^{\frac{SNR}{10}}\right) \quad \text{bits} \qquad (1)$$

2.2 Discrete equiprobable input signal

We model the discrete input signal as a Pulse Amplitude Modulation (PAM) signal with N symbols, the noise Z as independent and Gaussian distributed as in the continuous case. All symbols have the same probability $p(X=x_i)=1/N$, denoted by $p(x_i)$. This leads to the output signal distribution shown in Figure 2.

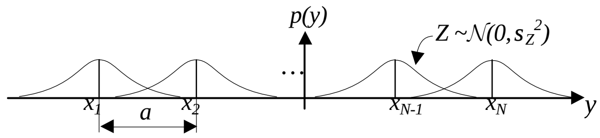

Figure 2: Discrete equiprobable signal using PAM.

Since symbols are equiprobable and the noise Z is independent, the values of the symbols x_i should be equidistant, in order to jointly minimize the error and the energy: $x_i = x_1 + (i\text{-}1)\times a$. Taking into account the energy constraint allows to determine x_1 and a:

$$\left.\begin{aligned} E\left[X^2\right] &= \sum_{i=1}^{N} x_i^2\cdot p\left(x_i\right) \le \sigma_X^2 \\ E\left[X\right] &= \sum_{i=1}^{N} x_i\cdot p\left(x_i\right) = 0 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x_1 &= -\sigma_X\sqrt{3\frac{N-1}{N+1}} \\ a &= 2\cdot\sigma_X\sqrt{\frac{3}{N^2-1}} \end{aligned}\right.$$

This leads to the capacity C_N for discrete equiprobable input signal with N symbols (Eq. 2):

$$\begin{aligned} C_N &= \sum_{i=1}^{N}\int_{y=-\infty}^{+\infty} p\left(y\,|\,x_i\right)p\left(x_i\right)\log_2\frac{p\left(y\,|\,x_i\right)}{p\left(y\right)}dy \\ p\left(y\,|\,x_k\right) &= \frac{1}{\sqrt{2\pi}\,\sigma_Z}e^{-\frac{\left(y-x_k\right)^2}{2\sigma_Z^2}} \\ p\left(y\right) &= \sum_{j=1}^{N}p\left(x_j\right)p\left(y\,|\,x_j\right) \end{aligned} \qquad (2)$$

The probability $p(y|x_i)$ is the probability that the continuous symbol y is recognized when the symbol x_i is sent. The variance σ_Z^2 of the noise Z is given by $\sigma_Z^2 = \sigma_X^2\cdot 10^{-SNR/10}$.

The capacity C_N has to be determined by numerical integration. Figure 3 compares the continuous capacity C (Eq. 1) and the discrete equiprobable capacity C_N (Eq. 2). There is an asymptotic difference of 1.53 dB between C and C_∞ (the discrete capacity for $N=\infty$), so called *shaping loss* (which is due to the use of an uniform probability density function instead of a Gaussian one). In all cases, the continuous capacity is greater than the discrete equiprobable capacity.

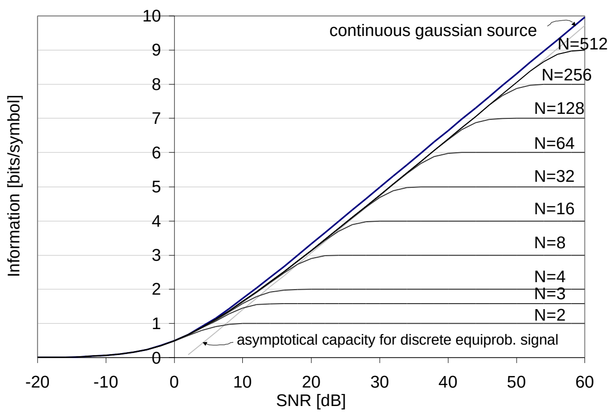

Figure 3: Comparison of the capacity for Gaussian input C and for discrete equiprobable input C_N as a function of the number of symbols N and of the SNR.

2.3 Discrete non-equiprobable input signal

In practical situations such as occurring in real BCIs, the input signals x_i are often not equiprobable ($p(x_i) \neq 1/N$). This leads to a capacity (in bits/symbol) approximately bounded by the continuous capacity C and by the entropy of the non-equiprobable source $H(X)$:

$$C_N \leq \min\left(\frac{1}{2}\log_2\left(1+10^{\frac{SNR}{10}}\right); H(X)\right) \quad (3)$$

Eq. 3 can be used to estimate the actual capacity when non-equiprobable input symbols are used. It shows that this capacity varies according to the ordering of the symbols: higher capacities are reached when the most likely symbols have the lowest energy. This clearly has consequences on the design of a BCI protocol, and ought to be taken into account. However, since we are here mainly concerned with an upper capacity bound estimation, we will in the sequel not further elaborate on this aspect.

3. BCI MODEL

The BCI is modelled as an additive white Gaussian noise (AWGN) channel (as in [35]), followed by a generic classifier (Figure 4). The input signal X models the brain, a memoryless source discrete of N possible tasks or mental states (classes, e.g. "relax", "left movement", "mental calculation") x_i, $i=1..N$, that are all supposed to have the same a priori probability $p(x_i)=1/N$. As in 2.2, the noise is independent from the input signal, so the N symbols are supposed to have equidistant amplitudes (situation depicted in Figure 2). The Gaussian noise Z models the noise induced by the background activity of the brain.

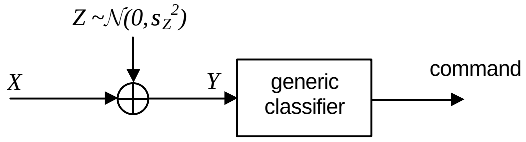

Figure 4: Model of the BCI using an AWGN channel.

The classifier recognizes M output mental states y_j, $i=1..M$ where $M=N$ for classifiers without rejection capability and $M=N+1$ for classifiers with rejection capability, leading to the equivalent lattice scheme shown in Figure 5. The NxM transition matrix $p(y_j|x_i)$, also called confusion matrix is computed during the classifier training phase. This matrix is composed of the probabilities that a mental state x_i is recognized as a mental state y_j, and its diagonal elements $p(y_i|x_i)$ are the classifier accuracies for each class [20], [25].

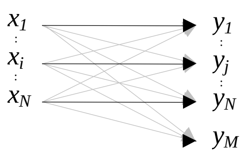

Figure 5: Equivalent lattice network for the BCI model shown on Figure 4. Black arrows depict the diagonal elements $p(y_i|x_i)$ and grey arrows depict the non-diagonal elements or classification error.

This BCI model however does not truly corresponds to a real BCI application, but more to an ideal BCI. First, the source is not always memoryless. For example, in average-trial protocols, the user repeats the same symbol a given number of times: the probability that the next symbol is the same than the current one is high. Another example is in text typing applications where the probability of the next letter or group of letter can be predicted with some accuracy. Secondly, the noise is not always Gaussian distributed. For instance, with a BCI that uses as features the energy in specific frequency bands, the noise is distributed according to a Rayleigh law. Thirdly, the symbols a priori probabilities are not always identical (e.g. [14], [25], [26]), this especially in applications where commands are infrequently emitted [2]. Last but not least, the fact that symbol values are most of the time imposed by the used feature might impose additional constraints on x_i.

Despite the fact that the proposed model does not truly correspond to a real BCI, it allows to define an approximate upper capacity bound C_M of the BCI. This upper capacity bound is therefore the one defined for discrete equiprobable inputs (cf. Eq. 2 and Figure 3) but with M instead of N symbols. Since C_M is calculated using numerical integration, which is not always practical, it can be written using the continuous capacity C and the entropy of a discrete source with M equiprobable symbols $\log_2 M$, leading to Eq. 4:

$$C_M = \min\left(\frac{1}{2}\log_2\left(1+10^{\frac{SNR}{10}}\right); \log_2 M\right) \quad (4)$$

This bound is also larger than or equal to the capacity bound for non-equiprobable symbols (Eq. 3) and thus can be considered as the “very upper bound” of the BCI capacity.

4. EXISTING BIT-RATE DEFINITIONS

The upper capacity bound obtained according to the BCI model from Section 3 can be compared to the bit-rate definitions used in the BCI domain. If this bound holds, every bit-rate definition should lead to a lower bit-rate than from the model.

Three bit-rate definitions are in use in the BCI domain. The first one is due to Farwell and Donchin in 1988 [12] when they designed the first BCI. The second definition is due to Wolpaw *et al* in 1998 [38], and the third one to Nykopp in 2001 [25]. All theses definitions are based on Shannon channel capacity. All bit-rates are indicated in bits/symbol, and can be converted to bits/minute according to:

$$B = V \times R \quad \text{[bits/minute]}$$

with V being the classification speed (in symbols/minute) and R the information carried by one symbol (in bits/symbols).

The most popular definition is the one from Wolpaw, which is reasonably simple and has often been used [37], [38], [39], [40]. The most generic definition is the one from Nykopp, which corresponds to Shannon channel capacity theory. Wolpaw’s as well as Farwell and Donchin’s definitions are in fact simplifications of Nykopp’s definition.

In order to compare our model with theses bit-rate definitions we assumed the use of Bayes hard-classifier, known to be the optimal hard-classifier if the underlying distribution of the data is known [11] (which is the case since Z is known, see Figure 2).

4.1 Nykopp definition

Nykopp's capacity has been introduced in the framework of the Adaptive Brain Interface (ABI) project [25]. The ABI is a BCI with rejection capacity meaning that no decision is taken if the confidence level of the classification does not exceed a certain threshold. This is modeled by means of an erasure channel where some symbols might be lost during transmission [7]. In summary, Nykopp's capacity is defined by:

$$C = \max_{p(x)} I(X;Y)$$
$$R_{Nykopp} = I(X;Y) = H(Y) - H(Y|X) \qquad (5)$$
$$H(Y) = -\sum_{j=1}^{M} p(y_j) \cdot \log_2 p(y_j)$$
$$p(y_j) = \sum_{i=1}^{N} p(x_i) \cdot p(y_j|x_i)$$
$$H(Y|X) = -\sum_{i=1}^{N}\sum_{j=1}^{M} p(x_i) \cdot p(y_j|x_i) \cdot \log_2 p(y_j|x_i)$$

The a priori symbol probabilities $p(x_i)$ are computed by means of the Arimoto-Blahut optimisation algorithm [17], in order to obtain the best bit-rate or capacity for the underlying channel specified by a given transition matrix. This does not truly correspond to a real BCI because in most cases, the a priori symbols probability is imposed by the application, e.g. [14], [25], [26].

4.2 Farwell & Donchin definition

When designing the first BCI in 1988, Farwell & Donchin introduced a bit-rate definition that did not take the classifier accuracy into account. The assumptions made were of a classifier without rejection ($M=N$), and perfect (i.e. no classification error). This therefore leads to an identity transition matrix $p(y_j|x_i)=I$ of size NxN. The mental states were assumed to be equiprobable, thus $p(x_i)=1/N$. From Eq. 5, these assumptions lead to the following bit-rate definition :

$$R_{Farwell\&Donchin} = \log_2 N \ \text{[bits/second]}$$

As the classifier is considered to be perfect, this measure does of course not correspond to a real BCI and should therefore not be used in practice. Its main interest is to show the maximum achievable bit-rate for high SNRs.

4.3 Wolpaw definition

In 1998, Wolpaw *et al* suggested that it could be interesting to consider the performance measurement not only from the accuracy point of view but also from the information rate point of view [38]. Using the definition of the information rate proposed by Shannon (see Eq. 5) for noisy channels, they made some simplifying assumptions.

First, they supposed that N symbols are recognized if N symbols are emitted by the user. They did not consider additional symbols (such as a "not recognized" mental state) as would be the case for classifiers with rejection, or their equivalent erasure channels. The second assumption is that the symbols (or mental states) all have all the same a priori occurrence probability $p(x_i)=1/N$. The third assumption is that the classifier accuracy P is the same for all target symbols ($p(y_j|x_i)=P$ for $i=j$))[1]. The fourth assumption is that the classification error $1-P$ is equally distributed amongst all remaining symbols ($p(y_j|x_i)=(1-P)/(N-1)$ for *i¹j*).

From Eq. 5, all theses assumptions lead to the following simplified bit-rate definition :

$$R_{Wolpaw} = \log_2 N + P \cdot \log_2 P + (1-P) \cdot \log_2 \frac{1-P}{N-1} \qquad (6)$$

5. RESULTS AND DISCUSSION

5.1 Bit-rates computation and signal-to-noise ratio estimation

Based on the analysis of a number of experimental published protocols and results, it has been possible to compute Wolpaw's and Nykopp's bit-rates in bits/trial and bits/minute (Table 1). The SNR has been graphically determined using Figure 8. For published results that are not specifying the transition matrix needed to compute Nykopp's bit-rate, the transition matrix was simulated using Bayes' classifier theory and the assumptions proposed in Section 2.2. These computations show that the typical BCI signal-to-noise ratio can be estimated to lie between –6 and 8 dB (see Table 1). An exception is for the first four BCIs of the Table where evoked potentials are used: these BCIs cannot really be compared to the others. The mean bit-rate $\overline{B}$ of all bit-rates B of Table 1 is only 9 bits/minute, which is very low compared to what is stated in [39] or compared to a typical keyboard bit-rate.

5.2 Determination of the theoretical optimal number of symbols

Using the same assumptions as in Section 3 (M-PAM feature, independent Gaussian noise and equiprobable symbols), the transition matrix is computed from Bayes classification theory [11] for varying numbers of symbols and SNRs. The classifier accuracy P is then determined as the maximum of the diagonal of the transition matrix and Wolpaw's bit-rate (Eq. 6) is computed, which leads to Figure 6. This figure shows that for typical BCI signal-to-noise ratios, the optimal number of symbols is 3 or 4, which corresponds to findings from studies on the optimization of the number of symbols [10], [23], [26].

[1] The mean or the maximum of the transition matrix diagonal is sometimes used when the classifier accuracy is not the same for all classes [26].

BCI group	N	$\overline{P}$	R_{Wolpaw}	R_{Nykopp}	SNR	$\overline{V}$	B
[6]+	10	90.0	2.54	2.51	17.5	10.8	27.4
[24]+	36	80.0	3.42	3.78	25.0	11.1	38.0
UIUC [9]*+	36	80.0	3.42	3.78	25.0	6.9	23.4
UIUC [12]*+	36	95.0	4.63	4.62	30.7	2.3	10.7
BerlinBCI [10]+	2	87.5	0.46	0.46	1.2	13.3	6.1
BerlinBCI [10]+	3	73.1	0.47	0.46	0.8	13.3	6.3
BerlinBCI [10]+	4	61.2	0.42	0.42	0.1	13.3	5.6
BerlinBCI [10]+	5	51.9	0.36	0.37	-0.5	13.3	4.8
BerlinBCI [10]+	6	44.8	0.31	0.34	-0.9	13.3	4.1
BerlinBCI [3]+	2	92.0	0.60	0.60	3	30.0	17.9
BerlinBCI [4]+	2	72.0	0.14	0.14	-4.7	120	17.3
[30]*+	4	48.0	0.18	0.17	-3.9	15.0	2.7
Graz-BCI [26]+	2	91.0	0.56	0.56	2.5	9.5	5.4
Graz-BCI [26]+	3	78.0	0.60	0.58	2.3	9.5	5.7
Graz-BCI [26]+	4	63.0	0.46	0.46	0.6	9.5	4.4
Graz-BCI [26]	5	52.7	0.38	0.43	-0.2	9.5	3.7
Graz-BCI [31]*+	2	85.6	0.41	0.40	0.5	15.0	6.1
Oxford [28]+	2	75.0	0.19	0.19	-3.4	9.0	1.7
Oxford [34]+	2	86.5	0.43	0.42	0.7	5.0	2.1
LIINC [27]+	2	73.0	0.16	0.16	-4.3	9.0	1.4
Wadsworth [22]*+	2	75.0	0.19	0.19	-3.4	15.0	2.8
Wadsworth [23]+	2	89.3	0.51	0.51	1.9	10.9	5.6
Wadsworth [23]+	3	77.9	0.60	0.57	2.2	10.9	6.6
Wadsworth [23]+	4	74.7	0.78	0.77	4.0	10.9	8.5
Wadsworth [23]+	5	67.0	0.75	0.77	3.8	10.9	8.2
ABI [25]	3	69.2	0.39	0.43	-0.6	NA	NA
ABI [20]	3	53.1	0.12	0.15	-5.8	11.0	1.3
ABI [33]	3	88.3	0.95	0.96	5.8	NA	NA
Neil Squire [5]*	2	98.9	0.91	0.11	7.2	NA	NA
Neil Squire [21]*	2	86.6	0.43	0.26	0.9	NA	NA
IM2 [15]*	3	59.4	0.21	0.20	-3.5	NA	NA
EPFL [13]*	3	80.4	0.68	0.68	3.1	6	4.1

Table 1: Computation of the bit-rates for various published BCIs according to Wolpaw's and Nykopp's definitions (R_{wolpaw} and R_{nykopp}, in bits/trial). The mean accuracy $\overline{P}$ (in %) and speed $\overline{V}$ (in trials/minute) are calculated for a given experiment taking all users into account. The SNR (in dB) is graphically determined using Figure 8. The bit-rate B is computed in bits/minute using R_{wolpaw} and $\overline{V}$ for the sake of comparison with others studies. An asterisk (*) denotes articles where the bit-rate definition was not specified; it is assumed in these cases that Wolpaw's definition can be used. A plus (+) denotes articles where the transition matrix was simulated to allow for Nykopp's bit-rate calculation. "NA" stands for "Not Available".

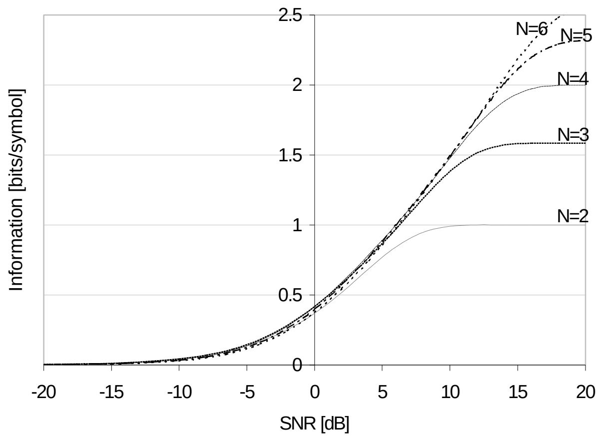

Figure 6: Bit-rates using Wolpaw's definition for SNRs and number of symbols specific to various BCI applications (see Table 1).

5.3 Discussion of Wolpaw's bit-rate definition

Wolpaw's definition (Eq. 6) does not hold in a number of practical situations. First, the number of recognized symbols is not always equal to the number of input symbols, like in the ABI of Millán *et al* where the classifier has a rejection capability [20], [25], [33].

Secondly, the a priori occurrence probability $p(x_i)$ is not always the same for all symbols, as has been shown in numerous studies [14], [16], [25], [26], [32]. This is especially true when using the oddball paradigm [1], [2], [9], [12], if the application is a virtual keyboard (due to unequal letter appearance frequencies [9], [12]), or if average-trial protocols are used [1], [9], [12], [18], [24] where the probability of the next symbol depends on the current symbol.

Thirdly, the classifier accuracy $p(y_i|x_i)$ has also been shown to differ between symbols [2], [5], [13], [20], [21], [25], [26], [29], [33]. Finally, the error is not equally distributed over the remaining symbols [2], [5], [13], [20], [21], [25], [26], [33].

Figure 7 compares the bit-rate R_{Nykopp} computed using Eq. 5 with the upper-bound capacity C_M defined by Eq. 4. In order to allow the comparison between the capacity established by our BCI model and the one provided by this definition, we made the hypothesis that all symbols are equiprobable; the Arimoto-Blahut algorithm was therefore not used. The small difference is due to the fact that a hard-classifier was used in Nykopp’s formalism.

Figure 8 presents Wolpaw's bit-rate for higher numbers of symbols and SNRs than Figure 6. In contradiction with the classical result from channel theory, Wolpaw's definition causes a decrease of the bit-rate when the number of symbols increases.

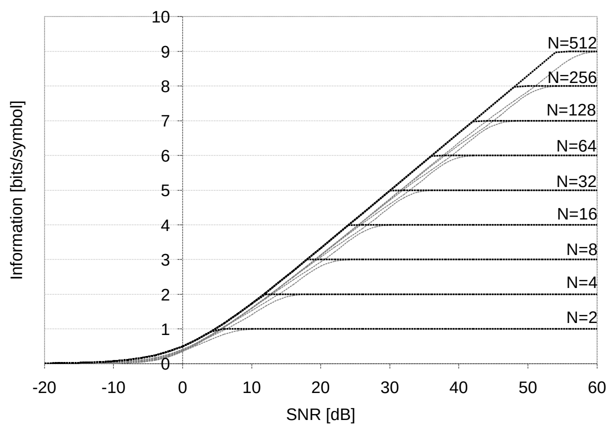

Figure 7: Comparison between Nykopp’s bit-rate definition R_{Nykopp} (Eq. 5) with hard-classifier (gray line) and the upper capacity bound (black lines) C_M (Eq. 4).

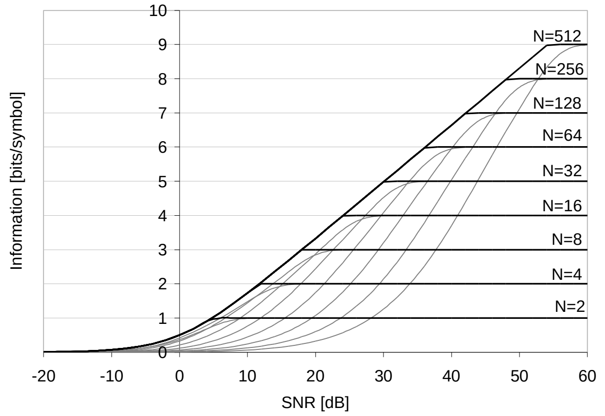

Figure 8 : Bit-rates using Wolpaw's definition (gray line, Eq. 6) and comparison with the upper capacity bound C_M (black line, Eq. 4).

Comparing Figure 7 and Figure 8, we can see that for some numbers of symbols and SNRs, Wolpaw's bit-rates are lower than Nykopp's bit-rates. Figure 9 presents the difference between Wolpaw's bit-rate and Nykopp's bit-rate $\Delta R = R_{Wolpaw} - R_{Nykopp}$. This figure shows values of N and of the SNR for which Wolpaw's bit-rates exceed or not Nykopp's rates. For N>5 symbols, Wolpaw's definition leads to bit-rates that are lower than those obtained according to Nykopp's definition (underestimation). For N=2, both definitions lead to the same bit-rate since the error is distributed on one symbol only. For N<5 and typical BCI signal-to-noise ratios, Wolpaw's bit-rates are slightly larger than Nykopp's bit-rates (less than 0.1 bits/trial). This dependence on N can also be verified on Table 1.

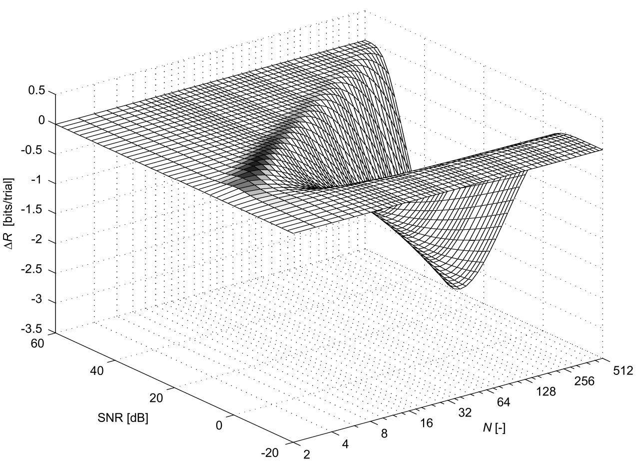

Figure 9: Difference ΔR between Wolpaw's bit-rates (Figure 8) and Nykopp's bit-rates (Figure 7), in bits/trial. ΔR values are positive when Wolpaw's bit-rates are larger than Nykopp's bit-rates and negative when lower. Since positive values are very small, they are marked in dark.

The underestimation in bit-rate from Wolpaw's definition is due to the assumption that the error is equally distributed over all remaining symbols. This is clearly not the case, since for M-PAM features with equiprobable occurrence probabilities, the error will in fact decrease with the distance between two symbols, as shown on Figure 10.

If Wolpaw's definition is used for assessing the performance of a particular BCI, the result will be that this BCI has a lower bit-rate than competitor BCIs assessed using Nykopp's definition, e.g. [6], [18], [26]. More importantly, if this definition is employed to determine the optimal number of symbols (like in [10] or [23], [26]), this could lead to wrong conclusions since for high number of symbols, Wolpaw's bit-rate will always be lower than with smaller number of symbols.

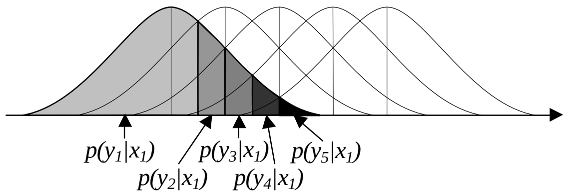

Figure 10 : The error decreases with the distance between M-PAM features. Example for $N=5$ with equidistant symbols and independent Gaussian noise.

6. CONCLUSIONS

In this paper, we have defined a model that leads to the definition of the upper capacity bound of a BCI. We also compared the capacity from this model with the bit-rate definitions commonly used in the BCI domain. It has been shown here that all the existing bit-rate definitions are below the capacity C_M from Eq. 4. This confirms the validity of C_M as an upper capacity bound for BCI applications.

The capacity and the bit-rate allow to make objective comparisons between BCIs that are using different protocols or that are designed for different applications. It can also help to solve the features selection problem [8]. Nevertheless, for applications where a high classifier accuracy is needed (e.g. control of a wheelchair or of a robot in hostile environments), the bit-rate may not be the ideal performance measure [37]. In such critical applications, bit-rate definitions including cost-functions could lead to a better measure, but with the disadvantage of losing some “objectivity” since the cost of a wrong decision would be application dependent.

Although the bit-rate is an objective measure, it does not take all the BCI parameters into account. For asynchronous BCIs where most of the time the user does not think to anything in particular (“idle state” [2]), the bit-rate will be lower than when using other BCI paradigms because of a higher $p(x_i)$ for that idle state. Also, for BCIs that use evoked related potentials, the bit-rate is generally higher than for “normal” BCIs because theses potentials allow for the use of more symbols. Such BCIs should thus not be compared with BCIs using other paradigms.

The study of several BCIs showed that their average bit-rate is of the order of 9 bits/minute (using either Wolpaw's or Nykopp's definition) and that the typical SNR is mainly between –6 to 8 dB. This allowed to determine the theoretical optimal number of symbols for Wolpaw's definition, which was found to be consistent with findings from other researchers.

Theoretical as well as practical comparisons between Wolpaw's and Nykopp's definitions show that Wolpaw's bit-rates are lower than Nykopp's bit-rates for more than 5 symbols. Wolpaw's definition should not be used since it underestimates Nykopp's measure. Instead, this latter one should be used, but without the Arimoto-Blahut optimization of the a priori symbols probability. As a number of BCIs currently use Wolpaw's bit-rate, a switch to Nykopp's bit-rate would make BCI performance comparisons difficult. Therefore, to allow comparison with previous studies and if feasible, bit-rates computed according to all possible definitions should be indicated.

ACKNOWLEDGEMENTS

This work is supported in part by the Swiss National Center of Competence in Research (IM)2, Interactive Multimodal Information Management (http://www.im2.ch/). The authors would also like to acknowledge fruitful discussions with Teodor Iulian Alecu.

REFERENCES

[1] J. D. Bayliss, *A Flexible Brain-Computer Interface (PhD Thesis)*, University of Rochester, Department of Computer Science, Rochester, New York, 2001.

[2] G.E. Birch, and S.G. Mason, "Brain-computer interface research at the Neil Squire Foundation", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No. 2, pp. 193-195, June 2000.

[3] B. Blankertz, G. Curio, and K.-R. Müller, "Classifying single trial EEG: Towards brain computer interfacing", In T. G. Diettrich, S. Becker, and Z. Ghahramani, editors, *Advances in Neural Inf. Proc. Systems* (NIPS 01), vol. 14, pp. 157-164, 2002.

[4] B. Blankertz, G. Dornhege, C. Schäfer, R. Krepki, J. Kohlmorgen, K.-R. Müller, V. Kunzmann, F. Losch, and G. Curio, "Boosting bit rates and error detection for the classification of fast-paced motor commands based on single-trial EEG analysis", *IEEE Trans. on Neural Systems and Rehab. Eng.*, Vol. 11, No. 2, 2003.

[5] Z. Bozorgzadeh, G.E. Birch, and S.G. Mason, "The LF-ASD brain computer interface: on-line identification of imagined finger flexions in the spontaneous EEG of able-bodied subjects", *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing* (ICASSP2000), Vol. 4, pp. 2385-2388, 2000.

[6] M. Cheng, D. Xu, X. Gao, and S. Gao, "Brain-Computer Interface with High Transfer Rates", *8th Int. Conference on Neural Information Processing (ICONIP2001)*, Shanghai, China, Nov. 14-18 2001.

[7] T. M. Cover, and J. A. Thomas, *Elements of Information Theory*, John Wiley & Sons Inc, 1991.

[8] A. Date, "An Information Theoretic Analysis of 256-Channel EEG Recordings: Mutual Information and Measurement Selection Problem", *3rd Int. Conf. on Independent Component Analysis and Blind Signal Separation* (ICA2001), San Diego, California, pp. 185-188, Dec. 9-12 2001.

[9] E. Donchin, K. M. Spencer, and R. Wijesinghe, "The Mental Prosthesis: Assessing the Speed of a P300-Based Brain–Computer Interface", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No. 2, pp. 174-179, June 2000.

[10] G. Dornhege, B. Blankertz, G. Curio, and K.-R. Müller, "Increase information transfer rates in BCI by csp extension to multi-class", *Advances in Neural Inf. Proc. Systems (NIPS03)*, Vol. 16, http://books.nips.cc/, 2003.

[11] R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern Classification, 2nd ed.*, John Wiley & Sons, Inc., Oct. 2000.

[12] L.A. Farwell, and E. Donchin, "Talking off the top of your head: toward a mental prosthesis utilizing event-related brain potentials", *Electroencephalography and Clinical Neurophysiology*, Vol. 70, pp. 510-523, 1988.

[13] G. N. Garcia Molina, T. Ebrahimi, U. Hoffmann, and J.-M. Vesin, "Direct brain-computer communication through EEG signals", Swiss Federal Institute of Technology (EPFL), To appear in the *IEEE EMBS Book Series on Neural Engineering*, 2003.

[14] C. Guger, H. Ramoser, and G. Pfurtscheller, "Real-Time EEG Analysis with Subject-Specific Spatial Patterns for a Brain–Computer Interface", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No. 4, pp. 447-456, Dec. 2000.

[15] E. Gysels, P. Celka, "Phase Synchronization for the Recognition of Mental Tasks in a Brain Computer Interface", submitted to *IEEE Trans. on Neural Sys. and Rehab. Eng.*, 2004.

[16] E. Haselsteiner, and G. Pfurtscheller, "Using Time-Dependent Neural Networks for EEG Classification", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No. 4, pp. 457-463, Dec. 2000.

[17] L. Ip, "The Blahut-Arimoto Algorithm for the Calculation of the Capacity of a Discrete Memoryless Channel", Technical Report, Berkeley Uni., Dec. 10 1999.

[18] J. Kronegg, T. Alecu, and T. Pun, "Information theoretic bit-rate optimization for average trial protocol Brain-Computer Interfaces", *Proc. HCI International 2003*, Crete, Greece, pp. 1437-1440, June 22-27 2003.

[19] J. Kronegg, S. Voloshynovskiy, T. Pun, "Upper capacity bound of a Brain-Computer Interface", submitted to *European Signal Processing Conf.* (EUSIPCO2004), Vienna, Austria, Sept. 7-10 2004.

[20] J. Lehtonen, *EEG-based Brain Computer Interfaces (M.Sc diploma)*, Helsinki University of Technology, Dep. of Electrical and Comm. Eng., May 3 2002.

[21] S.G. Mason, and G.E. Birch, "A brain-controlled switch for asynchronous control applications", *IEEE Trans. Biomed. Eng.*, Vol.47, No.10, pp.1297-1307, Oct. 2000.

[22] D. J. McFarland, L. M. McCane, S. V. David, and J. R. Wolpaw, "Spatial filter selection for EEG-based communication", *Electroencephalography and Clinical Neurophysiology*, Vol. 103, pp. 386-394, 1997.

[23] D. J. McFarland, W. A. Samacki, and J. R. Wolpaw, "Brain-computer interface (BCI) operation: Optimizing information transfer rates," *Biological Psychology*, Vol. 63, No. 3, pp. 237-251, July 2003.

[24] P. Meinicke, M. Kaper, F. Hoppe, M. Heumann, and H. Ritter, "Improving Transfer Rates in Brain Computer Interfacing: a Case Study", *Advances in Neural Inf. Proc. Systems (NIPS02)*, Vol. 15, S. Becker, S. Thrun and K. Obermayer eds., MIT Press, Cambridge (MA), pp. 1107-1114, http://books.nips.cc/, 2003.

[25] T. Nykopp, *Statistical Modelling Issues for The Adaptive Brain Interface (M.Sc diploma)*, Helsinki University of Technology, Dep. of Electrical and Comm. Eng., June 9 2001.

[26] B. Obermaier, C. Neuper, C. Guger, and G. Pfurtscheller, "Information Transfer Rate in a Five-Classes Brain-Computer Interface", *IEEE Trans. on Neural Sys. And Rehab. Eng.*, Vol. 9, No. 3, pp. 283-288, Sept. 2001.

[27] L. Parra, C. Alvino, A. Tang, B Pearlmutter, N. Yeung, A. Osman, and P. Sajda, "Linear Spatial Integration for Single Trial Detection in Encephelography", *NeuroImage*, Vol. 17, pp. 223-230, 2002.

[28] W.D. Penny, and S.J. Roberts, "Experiments with an EEG-based computer interface", Technical Report, *Dep. of Electrical Eng., Imperial College*, 1999.

[29] J. Perelmouter, and N. Birbaumer, "A binary spelling interface with random errors", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No 2, pp. 227-232, June 2000.

[30] J. B. Polikoff, H. T. Bunnell, and W. J. Borkowski Jr., "Toward a P300-based Computer Interface", *Proc. Rehab. Eng. and Assistive Tech. Soc. of North America* (RESNA'95), Arlington, Va, pp. 178-180, 1995.

[31] H. Ramoser, J.R. Wolpaw, and G. Pfurtscheller, "EEG-Based communication: Evaluation of alternative signal prediction methods", *Biomedizinische Technik*, Vol. 42, No. 9, pp. 226-233, 1997.

[32] H. Ramoser, J. Müller-Gerking, and G. Pfurtscheller, "Optimal Spatial Filtering of Single Trial EEG During Imagined Hand Movement", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No. 4, pp. 441-446, Dec. 2000.

[33] F. Renkens, and J. del R. Millán, "Brain-Actuated Control of a Mobile Platform", *Int. Conf. on Simulation of Adaptive Behavior, Workshop on Motor Control in Humans and Robots*, Edinburgh, Aug. 4-11 2002.

[34] S.J. Roberts, and W.D. Penny, "Real-time Brain Computer Interfacing: a preliminary study using Bayesian learning", *Medical and Biological Engineering and Computing*, Vol 38, No. 1, pp.56-61, 2000.

[35] M. Sahin, "Noise tolerance as a measure of channel discrimination for multi-channel neural interfaces", *23th Annual Int. Conf. of the IEEE Eng. in Med. and Biol. Soc.*, Istanbul, Turkey, Oct 25-28 2001.

[36] C. E. Shannon, "A Mathematical Theory of Communication", *The Bell System Technical Journal*, Vol. 27, pp. 379–423 and 623–656, July and Oct. 1948.

[37] T. M. Vaughan, W. J. Heetderks,L. J. Trejo, W. Z. Rymer, M. Wdnricli, M. M. Moore, A. Kubler. B. H. Oohkin, N. Birbaumer. E. Donchin, E. W. Wolpaw, and J. R. Wolpaw, "Brain-Computer Interface Technology: A Review of the Second International Meeting", *IEEE Trans. on Neural Sys. and Rehab. Eng.*, Vol. 11, No. 2, pp. 94-109, June 2003.

[38] J. R. Wolpaw, H. Ramoser, D. J. McFarland and G. Pfurtscheller, "EEG-Based Communication: Improved Accuracy by Response Verification", *IEEE Trans. on Rehab. Eng.*, Vol. 6, No. 3, pp. 326-333, Sept. 1998.

[39] J. R. Wolpaw, N. Birbaumer, W. J. Heetderks, D. J. McFarland, P. Hunter Peckham, G. Schalk, E. Donchin, L. A. Quatrano, C. J. Robinson, and T. M. Vaughan, "Brain-Computer Interface Technology: A Review of the First International Meeting", *IEEE Trans. on Rehab. Eng.*, Vol. 8, No 2, pp. 164-173, June 2000.

[40] J. R. Wolpaw, N. Birbaumer, D. J. McFarland, G. Pfurtscheller, and T. M. Vaughan, "Brain–computer interfaces for communication and control", *Clinical Neurophysiology*, Vol. 113, pp. 767–791, 2002.