

Laplacian channel state estimation for state dependent channels

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Abstract

In this paper, we extend the results for optimal transmission of the Gaussian channel state via the state-dependent channels to the communications of the Laplacian data. We derive a minimum mean square estimate (MMSE) of the stationary independent identically (i.i.d.) distributed Laplacian channel state corrupted by an additive white Gaussian noise (AWGN) and demonstrate that estimation accuracy is the same in both cases and does not depend on the channel state pdf. For transmission performance improvement we propose to decompose the Laplacian data using the paradigm of parallel source splitting. We show that for the case of infinite Gaussian mixture approximation of Laplacian source it is possible to significantly improve the performance of the considered communication protocol.

1 Introduction

The problem of information transmission via state-dependent channels has attracted significant attention in recent years. The main part of existing research has been focused on the information-theoretic analysis of fundamental performance limits of such protocols where the highest possible rate of reliable transmission was investigated. This problem has been studied by Gel'fand and Pinsker [1] for the general case of discrete memoryless channels and by Costa for a particular case of the Gaussian memoryless channels [2] and it was demonstrated that in the latter case it is possible to achieve the rate of transmission equivalent to the capacity of the ideal AWGN channel when a random binning strategy is used for codebook design.

Another important research issue concerns optimal communications through the state-dependent channels (or estimation from the channel output) of the conveyed state information. In addition to the data-hiding, where besides mentioned capacity demands channel state recovery also can be required in some particular applications [3], this aspect is getting more importance due to the necessity of upgrading and rehauling analog communications systems with digital transmission systems in broadcasting

of audio, images and video [4, 5, 6]. Moreover, recently some results have been reported demonstrating the efficiency of this paradigm in the application of the quality enhancement of printed/scanned documents using side information [7].

Theoretical fundamentals of the optimal design of the protocols targeting the highest accuracy of channel state estimation was recently considered by Sutivong *et al.* [8]. Their analysis was restricted to the Gaussian memoryless set-up and it was demonstrated that simple uncoded transmission leads to the optimal system performance in terms of channel state estimate distortions contrarily to the Costa information transmission optimized set-up [2]. Finally, it was shown that power sharing technique can be used in the general case when both message communication with some rate that is below the channel capacity and enhanced state communication are required.

Bounding the performance of any practical channel state communications system, the result in [8] should be revised for each particular case (meaning, for different state statistics) in order to evaluate the actual performance. Moreover, being fundamental, the i.i.d. stationary Gaussian assumption has restricted application in practice. For instance, it was multiply reported that statistical properties of real images independently on the considered domain have highly non-Gaussian character (see, for example, [9]).

Thus we can formulate the main goal of this paper to answer to the following question: what is the real channel state transmission system performance for the case when the channel state has some non-Gaussian distribution?

The paper is organized as follows. In Section 2 problem formulation of the channel state revealing at the decoder will be presented. Section 3 contains the distortion analysis of i.i.d. Laplacian channel state communications via the state-dependent channels. Some aspects of image modeling are considered in Section 4. The analysis of the non-stationary transmission of the i.i.d. Laplacian data is performed in Section 5. Finally, Section 6 concludes the paper.

Notations We use capital letters to denote scalar random variables X and small letters x to designate their realizations. Vector random variables and their realizations are denoted as X^N and x^n , respectively, where the superscript N is used to designate length- N vectors $x^N = [x[1], x[2], \dots, x[N]]^T$ with k^{th} element $x[k]$. We use $X \sim p_X(x)$ or simply $X \sim p(x)$ to indicate that a random variable X is distributed according to $p_X(x)$. The mathematical expectation of a random variable $X \sim p_X(x)$ is denoted by $E_{p_X}[X]$ or simply by $E[X]$. Calligraphic fonts \mathcal{X} denote sets $X \in \mathcal{X}$ and $|\mathcal{X}|$ denotes the cardinality of set \mathcal{X} . \mathbf{I}_N denotes the $N \times N$ identity matrix.

For performance evaluation purpose we also define the SIR as $SIR = 10 \log_{10} \frac{\sigma_W^2}{\sigma_X^2}$ and the SNR as $SNR = 10 \log_{10} \frac{\sigma_W^2}{\sigma_Z^2}$ where σ_X^2 , σ_W^2 , σ_Z^2 represent the variances of the channel state, encoder output and noise, respectively.

2 Problem formulation

The generalized problem of joint information and channel state communications across state-dependent channels (Figure 1) can be formulated in the following way. A message m that is uniformly distributed over $\mathcal{M} = \{1, 2, \dots, \bar{M}\}$, $\bar{M} = 2^{NR}$, is encoded with rate $R = \frac{1}{N} \log_2 \bar{M}$ using side information $X^N \in \mathcal{X}^N$ about the channel state. The output has the following distribution:

$$p(y^N | w^N, x^N) = \prod_{i=1}^N p(y_i | w_i, x_i).$$

The decoder, given the channel output Y^N is attempting at both estimating \hat{m} that was sent and recovering \hat{X}^N with bounded average distortion. The rate-distortion pair (R, D) is said to be achievable if there exists such a $(2^{NR}, N)$ code defined by the following encoder mapping $W^N : \{1, 2, \dots, 2^{NR}\} \times \mathcal{X}^N \longrightarrow \mathcal{W}^N$ and decoder mappings

$\hat{m} : \mathcal{Y}^N \rightarrow \{1, 2, \dots, 2^{NR}\}$ and $\hat{X}^N : \mathcal{Y}^N \rightarrow \hat{\mathcal{X}}^N$ that $\frac{1}{2^{NR}} \sum_{i=1}^{2^{NR}} \Pr[\hat{M} \neq i | M = i] \rightarrow 0$ as $N \rightarrow \infty$ and $E[d^N(X^N, \hat{X}^N)] \leq D$, where $d^N(X^N, \hat{X}^N) = \frac{1}{N} \sum_{i=1}^N d(x_i, \hat{x}_i)$ and $d(x_i, \hat{x}_i) : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}^+$.

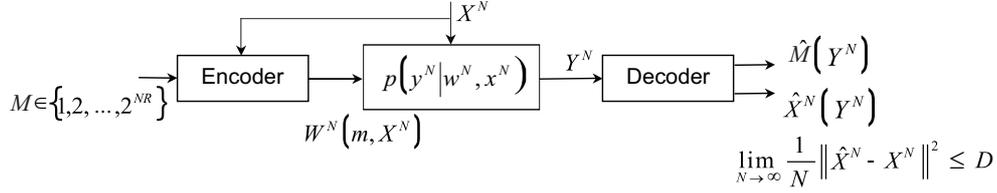


Figure 1: Generalized communications framework via the state-dependent channel.

The detailed analysis of the Gaussian formulation of this set-up can be found in [8].

2.1 Channel state estimation for state-dependent Gaussian channels

Basic set-up for the analysis of the particular case of the above formulated generalized problem when $R = 0$ and all data are assumed to be i.i.d. Gaussian is presented in Figure 2.

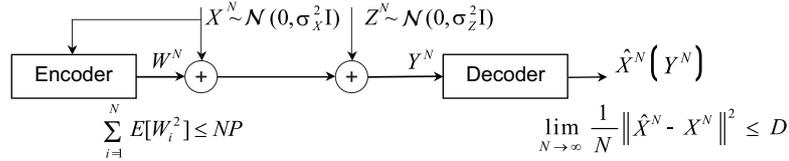


Figure 2: Channel state estimation in the state-dependent Gaussian channel.

This set-up was analysed in [8] and it was demonstrated that the highest quality estimate of the channel state \hat{X}^N can be obtained using uncoded transmission principle, meaning $W^N = \alpha X^N$, where $\alpha = \sqrt{\frac{P}{\sigma_X^2}}$, and decoder, given $Y^N = W^N + X^N + Z^N$, performs the MMSE estimation of interference in the following form:

$$\hat{X}^N = \frac{\sigma_X^2 + \sqrt{P\sigma_X^2}}{(\sigma_X + \sqrt{P})^2 + \sigma_Z^2} Y^N \quad (1)$$

with the average distortion D given by:

$$D = \frac{\sigma_X^2 \sigma_Z^2}{(\sigma_X + \sqrt{P})^2 + \sigma_Z^2}. \quad (2)$$

The corresponding performance of the Gaussian channel state estimation at the output of the Gaussian state-dependent channel can be found in Figure 3.

3 Channel state estimation for state-dependent channels: Laplacian case

As it was already pointed out in Section 1, the considered Gaussian set-up has restricted practical application due to the stochastic properties of real-world data and necessity to adopt the decoder (the MMSE estimator) to the particular channel state statistics.

The main goal of this section is to evaluate the optimal channel state communications system performance designed according to the set-up presented in Figure 2 when X^N is i.i.d. Laplacian:

$$f(x) = \frac{\lambda}{2} e^{-\lambda|x|}, \quad (3)$$

where λ is the distribution parameter. As the motivation for this particular model selection we used its simplicity and accuracy for approximation of global statistics of the coefficients in wavelet and discrete cosine transform subbands [10] that was successfully exploited in lossy image compression [11] as well as for the denoising of images corrupted by the AWGN [12].

Assuming that W^N is obtained as a scaled version of the realization of the channel state X^N , $W^N = \alpha X^N$, $\alpha = \sqrt{\frac{P}{\sigma_X^2}}$, one can obtain the following MMSE estimate \hat{X}^N based on the channel output $Y^N = X^N + W^N + Z^N$:

$$\begin{aligned} \hat{X}^N &= \left(\left(\frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{Y^N + \frac{\lambda \sigma_Z^2}{a}}{\sigma_z \sqrt{2}} \right) \right) \exp \left(-\frac{\lambda Y^N}{a} + \frac{\lambda^2 \sigma_Z^2}{2a^2} \right) + \exp \left(\frac{\lambda Y^N}{a} + \frac{\lambda^2 \sigma_Z^2}{2a^2} \right) \times \right. \\ &\left. \left(\frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{Y^N + \frac{\lambda \sigma_Z^2}{a}}{\sigma_z \sqrt{2}} \right) \right) \right)^{-1} \left(\left(\frac{Y^N}{a} - \frac{\lambda \sigma_Z^2}{a^2} \right) \exp \left(-\frac{\lambda Y^N}{a} + \frac{\lambda^2 \sigma_Z^2}{2a^2} \right) \times \right. \\ &\left. \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{Y^N - \frac{\lambda \sigma_Z^2}{a}}{\sigma_z \sqrt{2}} \right) \right) + \left(\frac{Y^N}{a} + \frac{\lambda \sigma_Z^2}{a^2} \right) \exp \left(\frac{\lambda Y^N}{a} + \frac{\lambda^2 \sigma_Z^2}{2a^2} \right) \left(\frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{Y^N + \frac{\lambda \sigma_Z^2}{a}}{\sigma_z \sqrt{2}} \right) \right) \right), \end{aligned}$$

where $a = 1 + \alpha = 1 + \sqrt{\frac{P}{\sigma_Z^2}}$ and $\operatorname{erf}(x)$ denotes the error function, $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$.

The average distortions can be obtained by the following expectation:

$$D_{MMSE}^{Laplace} = \int_{\mathcal{X}} \int_{\mathcal{Z}} (x - \hat{x})^2 f_X(x) f_Z(z) dz dx, \quad (4)$$

where $f_X(x)$ is the Laplacian pdf of the channel state (3) and $f_Z(z)$ is the pdf of the channel that is assumed to be i.i.d. zero-mean Gaussian with the variance σ_Z^2 .

The expression for $D_{MMSE}^{Laplace}$ does not exist in the closed analytic form and (4) was evaluated using numerical integration. The obtained results are presented in Figure 3 versus performance of the state-dependent Gaussian channels for two different variances of X^N .

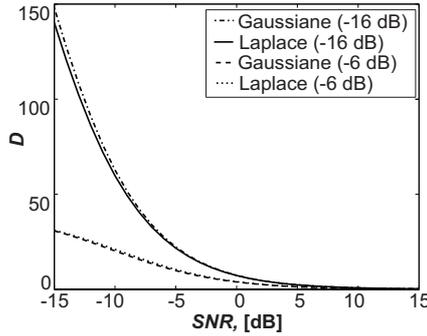


Figure 3: Estimation accuracy in terms of average distortion of the Gaussian and Laplacian channel states at the output of the state-dependent channel for two different SNRs.

These results allow to conclude that estimation of the Laplacian channel state, especially for the case of its smaller variance and at high SNR, is performed asymptotically

with the same accuracy as in case of the Gaussian set-up. Thus, the goal of the following sections is to find the answer to the following question: is it possible to enhance i.i.d. Laplacian state estimation performance from the output of the state-dependent channel?

4 Parallel splitting of Laplacian source

As it was already mentioned in the previous Section, Laplacian model was successfully exploited in both image compression and denoising, even more gain can be obtained from the local rather than global consideration of the data in wavelet subbands [13]. The corresponding procedure of local data samples classification based on their statistical properties is known as a *source splitting* [14] and establishes the mathematical relationship between local and global stochastic models using the *infinite Gaussian mixture model*. According to this model, the global zero mean Laplacian pdf (3) can be equivalently represented as a weighted mixture of zero-mean Gaussian pdfs with non-stationary variance that capture local data statistics:

$$p_X(x) = \int_0^\infty p_{X|\Sigma_X^2}(x|\sigma_X^2) p_{\Sigma_X^2}(\sigma_X^2) d\sigma_X^2, \quad (5)$$

where $p_{X|\Sigma_X^2}(x|\sigma_X^2) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{x^2}{2\sigma_X^2}}$ and $p_{\Sigma_X^2}(\sigma_X^2) = 0.5\lambda^2 e^{-0.5\lambda^2\sigma_X^2}$.

Therefore, this data can be simultaneously considered to be locally zero-mean Gaussian with the variance distributed according to the exponential pdf and at the same time having the Laplacian global statistics (Figure 4).

It is important to note that the state-of-the-art image compression algorithm known as *estimation-quantization* [15] is based on this model. In fact, omitting the practical details of communications of the local variances to the decoder, Hjørungnes, Lervik and Ramstad [14] were the first who theoretically demonstrated that the rate gain between Laplacian and Mixture Gaussian models can be as much as 0.312 bits/sample for high-rate compression regime.

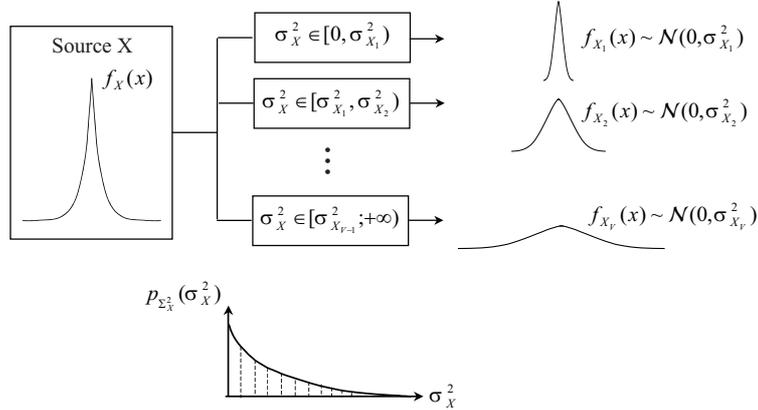


Figure 4: Source splitting as V parallel Gaussian channels.

Assuming the Ramstad set-up (availability of local variances at the decoder) in the application of the i.i.d. Laplacian host communications across the state-dependent Gaussian channels, we would like to investigate the influence of this side information on the accuracy of estimation.

5 Non-stationary transmission of the stationary Laplacian source via state-dependent channels

Problem formulation of communications of the i.i.d. Laplacian host X^N that is perfectly available at the encoder via the state-dependent channels when the information about local variances Σ_X^2 is available at the decoder is presented in Figure 5.

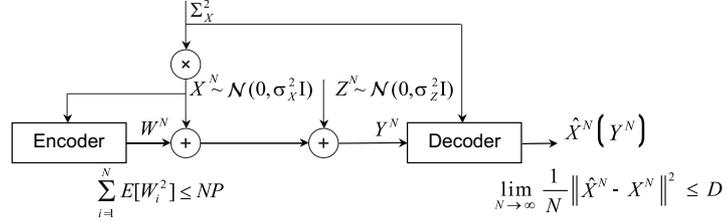


Figure 5: Non-stationary Laplacian host communications via the state-dependent channels.

According to the non-stationary Gaussian assumption that is equivalent to the globally stationary Laplacian data consideration, the optimal per-sample transmission of the host data can be performed using the uncoded transmission in the state-dependent Gaussian channels (Figure 2). The optimality of this protocol can be proved using Theorem 1 from [8]. Thus, to bound the average distortions, the following Lemma can be formulated.

Lemma 1. Consider a state dependent channel $Y^N = W^N(X^N) + X^N + Z^N$ with non-causal side information X^N globally distributed according to i.i.d. Laplacian distribution with parameter λ available at the encoder, independent noise Z^N , $Z_i \sim \mathcal{N}(0, \sigma_Z^2)$, the encoder power constraint $\frac{1}{N} \sum_{i=1}^N E[W_i^2] \leq P$ and side information (local variances, $\Sigma_X^{2N} = [\sigma_{X_1}^2, \sigma_{X_2}^2, \dots, \sigma_{X_N}^2]^T$, T denotes matrix transposition operation) perfectly available at the decoder. The MMSE error of the state X^N at the decoder is given by the following expectation:

$$D_{MMSE}^{Ramstad} = \int_0^\infty \frac{\sigma_X^2 \sigma_Z^2}{(\sigma_X + \sqrt{P}) + \sigma_Z^2} p_{\Sigma_X^2}(\sigma_X^2) d(\sigma_X^2) = \int_0^\infty \frac{\sigma_X^2 \sigma_Z^2}{(\sigma_X + \sqrt{P}) + \sigma_Z^2} \frac{\lambda^2}{2} e^{-\frac{\lambda^2 \sigma_X^2}{2}} d(\sigma_X^2). \quad (6)$$

Proof. The proof is based on the same arguments exploited in [8] for per-symbol transmission and the final result is obtained by the expectation with respect to the given prior local variance distribution.

To validate the performance of the proposed non-stationary Laplacian host communications across the state-dependent channel with stationary i.i.d. Gaussian noise we performed a number of experiments summarized in Figure 6 where the estimation accuracy of the non-stationary set-up is given versus the results of Section 3.

They demonstrate that significant performance improvement of the classical system can be obtained when it is adopted to the non-stationary local behavior of globally stationary Laplacian data especially at SNR below 0 dB. For quantitative comparison purpose, we selected performance of both systems in terms of distortions for the $SNR = -5$ dB. The correspondent values of D are: stationary case, $SIR = -16$ dB: $D = 41$, $SIR = -6$ dB: $D = 36.5$; stationary case, $SIR = -16$ dB: $D = 33.6$, $SIR = -6$ dB: $D = 33$ that proves the advantages of the proposed transmission paradigm.

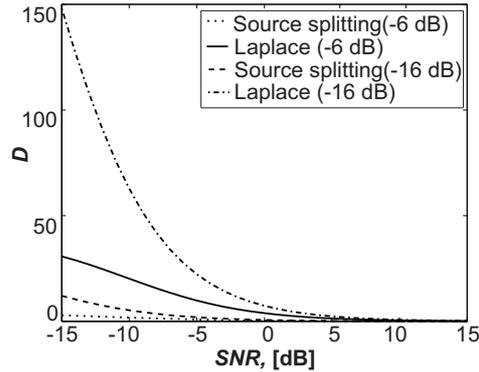


Figure 6: Adaptive Laplacian host communications via the state-dependent channels.

6 Conclusions

In this paper we considered the problem of i.i.d. Laplacian host communications via the state-dependent channel. We derived the decoder structure (the MMSE) estimate when the transition channel pdf is stationary zero-mean Gaussian with known variance and demonstrated that estimation of stationary i.i.d. Laplacian and Gaussian data is performed with similar accuracy. To enhance the accuracy of Laplacian host estimation we proposed side-information aided set-up that is based on the parallel Gaussian splitting of Laplacian source. In this set-up we assumed the realization of the host data to be perfectly available at the encoder and local variance of local Gaussian pdfs approximating the actual Laplacian to be available at the decoder. We derived the estimate for the best achievable estimation performance and performed a number of simulations that allowed us to conclude about significant performance improvement of the proposed system versus the classical one.

Despite high efficiency of Laplacian host non-stationary communications system, auxiliary channel for side information transmission should be used. In order to relax the requirements to this channel information-theoretic characteristics, we would like to extend the considered in this paper set-up to the case when non-perfect side information is available at the decoder [16].

Another possible research line consists in the current setup extension to the case when the local variances are available at both encoder and decoder and encoder optimization to extra auxiliary information available.

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