

**ADAPTIVE APERTURE FORMATION MATCHED WITH RADIOMETRY IMAGE  
SPATIAL SPECTRUM**

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*Abstract - The new approach to the radiometry image formation based on the matching of image characteristics with aperture synthesis is proposed. Quantitative analyze and comparison with conventional method of radiometry imaging are performed.*

**I. Introduction.**

Remote sensing systems in general and radiometry imaging systems in particular play an important role in modern information gathering system. The further development of these systems requires the special types of antennas in which small size and weight must be matched with high spatial resolution [1].

Antenna takes an notable place in the image formation systems because its parameters determine properties of the whole system. Antenna must estimate the image spatial spectrum until the frequency defined by the ratio of physical antenna size to wavelength. Therefore, the aim of this paper is to investigate the main peculiarities of antenna and object image structure matching based on the assumption that antenna is the spatial frequency filter.

Image formation process could be written according to the observation equation that is the Fredholm equation of the first kind with difference kernel, i.e. convolution

$$g(\theta, \varphi) = \iint_{\Omega} h(\theta - \theta', \varphi - \varphi') \cdot f(\theta', \varphi') d\theta' d\varphi' + n(\theta, \varphi) \quad (1)$$

where  $\theta, \varphi$  are spatial coordinates of space point;  $\theta', \varphi'$  are coordinates, defined the orientation of directional antenna pattern;  $g(\cdot, \cdot)$  is obtained image distorted by directional antenna pattern and noise;  $h(\cdot, \cdot)$  is magnitude directional antenna pattern;  $f(\cdot, \cdot)$  is original image;  $n(\cdot, \cdot)$  is noise conditioned by propagation channel and receiving part of radiometry imaging system;  $\Omega$  is space region for the spatial scanning.

Equation (1) is 2D convolution, that, using transform into spatial frequency domain, could be written as

$$\tilde{G}(m, p) = \tilde{H}(m, p) \cdot \tilde{F}(m, p) + \tilde{N}(m, p), \quad (2)$$

where  $\tilde{G}(m, p)$ ,  $\tilde{H}(m, p)$ ,  $\tilde{F}(m, p)$  and  $\tilde{N}(m, p)$  are the Fourier transforms from  $g, H, f$  and  $n$ , respectively.  $m$  and  $p$  are samples of spatial frequencies. The analysis of Eq. 2 makes possible to conclude that the quality of the observed image  $\tilde{G}(m, p)$  is determined by the matching of antenna spectrum  $\tilde{H}(m, p)$  with image spectrum  $\tilde{F}(m, p)$  under noise  $\tilde{N}(m, p)$  absence condition.

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## II. Proposed approach.

Investigations of the number of typical radiometry image spatial spectra show some peculiarities in the location of the image spatial spectrum harmonics. Spatial spectrum (Fig. 1,b) is calculated as Fourier transform from intensity distribution of original image (Fig. 1,a). As it shown in Fig. 1,b, image spatial spectrum harmonics have the specific placement, i.e. they locate along the axes that are shifted on the certain angle from the coordinate axes. Therefore, the main principle of image and antenna spatial spectra matching consists in antenna geometry finding whose spatial spectrum has the same structure as the image one [2]. In ideal case antenna must provide the estimation of all harmonics in the image spatial spectrum. However, radiometry imaging system spectrum is known to be more narrow than image one and the above condition satisfaction requires the large size aperture, the use of which is not expedient in a lot of applications [3]. Therefore, under this restriction, system with maximum information harmonic receiving will be preferred. Moreover, selective peculiarity of matched antenna spatial spectrum will decrease the noise influence whose harmonics have uniform spatial distribution in general. It is clear, that there are some difficulties in necessary antenna characteristics formation for continuum aperture, so, this approach would be more suitable for radiometry imaging system based on sparse antenna arrays [3].

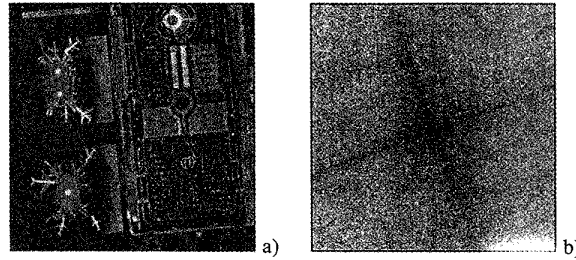


Fig. 1. Original image (a) and its spatial spectrum (b).

Lets write the main equation characterized the system with this type of antennas. The initial data for the antenna array is information about the current (field) distribution in the antenna aperture. Directional antenna pattern of antenna array could be defined as:

$$\dot{H}(\theta, \varphi) = \dot{H}_0(\theta, \varphi) \cdot \dot{H}_\Sigma(\theta, \varphi) \quad (3)$$

$$\text{where } \dot{H}_\Sigma(\theta, \varphi) = \iint_S \dot{I}_{p,q} \cdot \exp\{-j\beta(z_p(x) \cos\theta + z_q(y) \cos\varphi)\} dx dy \quad (4)$$

$\dot{H}_0(\theta, \varphi)$  is directional antenna pattern of a single antenna array element;  $\dot{H}_\Sigma(\theta, \varphi)$  is system multiplier, determined by aperture geometry and type of current (field) distribution;  $\dot{I}_{p,q}$  is field distribution in antenna array aperture;  $z_p, z_q$  are distances to the  $[p, q]$  element of the antenna array and  $x, y$  are coordinates in aperture  $S$ ;  $\beta = 2\pi/\lambda$  is wave number.

Since, directional pattern of antenna array has small dependence from directional pattern of the single element due to the fact that antenna elements with low directional gain are usually used, the next assumption could be made  $\dot{H}_0(\theta, \varphi) = 1$  and

$$\dot{H}(\theta, \varphi) = \dot{H}_{\Sigma}(\theta, \varphi). \quad (5)$$

In radiometry imaging system the power directional pattern is primer important characteristic and defined as

$$h(\theta, \varphi) = |\dot{H}(\theta, \varphi)|^2. \quad (6)$$

Spatial spectrum of antenna array is the Fourier transform from the normalized power directional pattern

$$\tilde{H}(m, p) = \mathfrak{F}\{h(\theta, \varphi)\}, \quad (7)$$

where  $\mathfrak{F}\{\cdot\}$  denotes the Fourier transform.

### III. Computer simulation.

To demonstrate the main particularities simulation of the above mentioned characteristics were accomplished based on two types of antenna array geometries: conventional antenna array with the square aperture (Fig. 2,a) and “cross-like” antenna array (Fig. 2,b) with the same number of antenna elements in the both cases. Fig. 3 represents the directional antenna patterns and Fig. 4 the spatial spectra of the corresponded geometries.

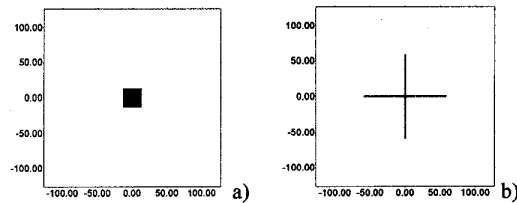


Fig. 2. Antenna array geometries: a) with square aperture; b) “cross-like”.

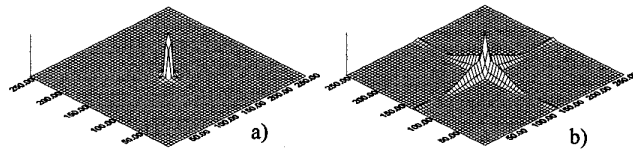


Fig. 3. Directional antenna patterns of the square aperture(a) and “cross-like” (b) antenna arrays.

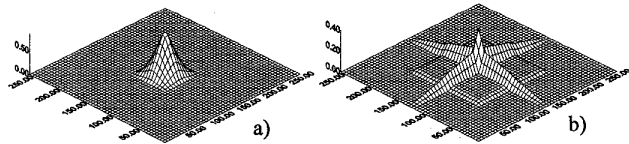


Fig. 4. Spatial spectra of the square aperture(a) and “cross-like” (b) antenna arrays.

Distinctive peculiarity of the “cross-like” antenna array is the spatial spectrum harmonics localization along the axes which have the same directions as the placement of antenna elements in the aperture, i.e. the close link between positions of antenna elements and spatial spectrum harmonics situation exists and it could be described as an autocorrelation function from the current (field) distribution

$$|\tilde{H}(m, p)|^2 = \tilde{I}_{p,q} \oplus \tilde{I}_{p,q}, \quad (8)$$

where  $\oplus$  denotes the autocorrelation function.

Based on the above mentioned arguments matching of antenna array with image spatial spectrum was performed. Since, the spatial spectrum of image, presented in Fig. 1,a is rather complicated, simple model object image on the constant background (Fig. 5,a) is used for the idea confirmation. This case is demonstrative for the radiometry imaging systems.

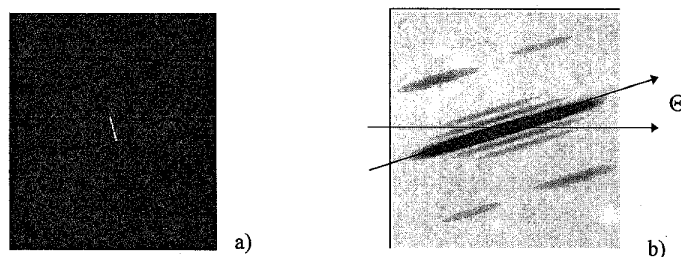


Fig. 5. Model object image (a) and its spatial spectrum.

For the matched antenna array design one part of the “cross-like” antenna array was used. Taking into account the link between antenna element and spatial spectrum harmonics localization, antenna array elements must be situated along the axis shifted on the angle  $\Theta$  according to the shift angle of image spatial spectrum harmonics (Fig. 5,b). Aperture geometry of the matched antenna array is shown in Fig. 6,a. Directional antenna pattern and spatial spectrum are given in Fig. 6,b and Fig. 6,c, respectively.

For both methods of radiometry image formation comparison: conventional (by means of antenna array with square aperture (Fig. 2,a)) that is not matched with image spatial spectrum method and proposed (by means of matched aperture (Fig. 6,a)) method quantitative analyze was performed. Comparison was made based on  $l_1$ - and  $l_2$ -norms of image formation.

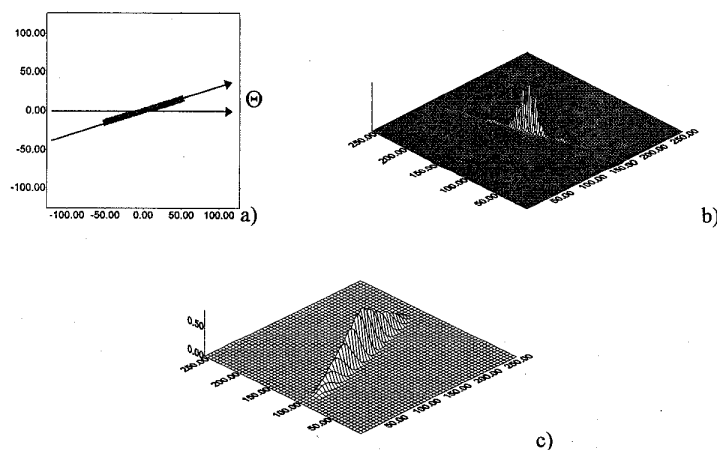


Fig. 6. Antenna array aperture matched with object (a), directional pattern (b) and spatial spectrum (c).

In  $l_1$ -norm image formation error could be written as:

$$E_1 = \frac{\sum_{i=1}^{256} \sum_{j=1}^{256} |g(i,j) - f(i,j)|}{\sum_{i=1}^{256} \sum_{j=1}^{256} |f(i,j)|}, \quad (8)$$

where  $g(i,j)$  is obtained image and  $f(i,j)$  is original image.

Mean square error or  $l_2$ -norm image formation error can be calculated according to the next equation:

$$E_2 = \frac{\sum_{i=1}^{256} \sum_{j=1}^{256} (g(i,j) - f(i,j))^2}{\sum_{i=1}^{256} \sum_{j=1}^{256} f^2(i,j)}. \quad (9)$$

Calculation results are presented in Table 1.

TABLE 1. Comparison of image formation errors in  $l_1$ - and  $l_2$ -norms.

Error type	Model image (Fig. 5,a)		Real Image (Fig. 1,a)	
	$E_1$	$E_2$	$E_1$	$E_2$
Conventional method	79.4%	56.7%	87.3%	68.1%
Proposed method	59.7%	20.1%	79.4%	40.1%

As it is shown in Table 1, proposed method makes possible to decrease the  $l_1$ -norm error in 1.3 times and  $l_2$ -norm error in 2.8 times for model image and in 1.1 and 1.7 times respectively for real image that confirms the proposed method advantage. Worse results for the real image caused by complicity of the real image spatial spectrum that requires more complicate array geometries than considered above.

#### IV. Conclusions.

In this paper new approach to radiometry image formation is presented. Comparison of two methods: conventional and proposed is made by means of quantitative analyze of  $l_1$ - and  $l_2$ -norm image formation errors shows advantages of the described method. The further investigation will be directed on the optimal estimation of image spectrum components orientation.

#### V. References:

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