HIGH RESOLUTION RADAR IMAGING SYSTEMS
WITH ROBUST IMAGE RESTORATION

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Abstract
The paper presents a robust approach to image restoration that combines the properties of classical regularized iterative algorithms and robust features based on the concept of M-estimators. The approach takes into consideration the peculiarities of radar image formation to provide optimal noise removal and image restoration including suppression of ringing, and preservation of edges and fine details.

1. Introduction
Modern remote sensing systems are widely used for the solution of a variety of applied radar problems. However, the quality of primary radar and radiometry images is far from desirable for the purposes of unequivocal data interpretation and object recognition. Among the main factors, that considerably aggravate image quality, are the spatial blurring effects of imaging systems and noise of different types. Spatial image blurring is caused by factors which depend on the particularities of observation. The main blurring factors, that lead to ambiguity of measurement, are finite imaging aperture, spatial atmospheric turbulence, sparseness of aperture resulting in large side lobes in a directional antenna pattern and nonoptimal beamforming caused by phase errors in aperture. Additionally, the situation is complicated by errors of measurement or noise in the receiver and registration equipment, which can be modeled by additive Gaussian noise. On the other hand, an additional problem arises due to the coherent nature of observation in such areas as radar imaging, SAR and ultrasonic imaging, that lead to the well-known speckle effect. In many cases, the received data is nonuniformly sampled owing to failures in the spatial scanning system or the physical nature of possible spatial sampling, resulting in information loss. The last two distortions can be represented by multiplicative impulse noise on the basis of the “salt and pepper” model.
For this reason, it appears to be necessary to provide additional measurements, which are either too expensive or require too much time or can not be accomplished at all due to the speed of the processes being studied or alteration of operating conditions. The alternative approach is characterized by less expenditure on additional measurement and corresponding time, and consists in mathematical compensation for the degradation using image restoration and noise suppression methods.

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The aim of the paper is to demonstrate the advantages of robust estimation strategies in radar image restoration over traditional restoration techniques based on Tikhonov regularization which is characterized by global image smoothing and low efficiency in the case of impulse noise presence.

2. Problem formulation
In the general case of image degradation caused by the above mentioned blurring factors and image distortion caused by Gaussian and impulse (mixed) noise, the following model is used

\[ g = \begin{cases} 
    s + n = Hf + n, & \text{with probability } 1 - p \\
    n_{imp}, & \text{with probability } p 
\end{cases} \]  

(1)

where \( g \) represents the degraded image, \( s \) is the signal component of the measurement and \( n \) represents the corresponding Gaussian noise component. \( H \) represents the linear spatially invariant blurring operator formed on the basis of the antenna beam scanning law and \( f \) denotes the original image. \( n_{imp} \) is impulse noise which occurs with probability \( p \) and

\[ n_{imp} = \begin{cases} 
    g_{\min}, & \text{with probability } 1 - q \\
    g_{\max}, & \text{with probability } q 
\end{cases} \]

where \( g_{\min} \) and \( g_{\max} \) are the minimum and maximum values of the dynamic range of the image and \( q \) denotes the occurrence probability of minimum and maximum values. In our model we assume that \( g_{\min} \) and \( g_{\max} \) have equal probability of occurrence.

3. Problem solution
The solution of the high quality imaging problem in the context of a restoration strategy consists in the estimation or retrieval of the original image starting from the degraded image using available \textit{a priori} information about the model of observation (1), knowledge of the blur function and statistics of noise and original image given in the form of image smoothness, solution constraints on some specific image features, and parametric models. This refers to the solution of inverse problems which are known to be ill-posed ones. Among the large number of linear and nonlinear methods developed for the solution of estimation problems we have chosen here the approach based on the theory of M-estimators due to its robust features. Two approaches are considered in the formulation of the robust radar image estimation problem. The first approach refers to the prefiltering strategy and the second one to robust residual estimation.

3.1. Robust restoration based on prefiltering
The main idea of this kind of robust restoration is to reduce the influence of noise factors on the later stages of image restoration by introducing the smallest degradation possible. Mathematically, it could be written according to the following minimization problem
\begin{equation}
\min_{\alpha} \mathcal{D}(f, \alpha) = \|g - Hf\|^2 + \alpha \|Cf\|^2
\end{equation}

\begin{equation}
\min_{\hat{g}} \mathcal{G}(\hat{g}) = \rho_\alpha(g - \hat{g})
\end{equation}

where \(|\cdot|\) is matrix norm, \(\alpha\) is a regularization parameter, \(\rho_\alpha(\cdot)\) is robust objective estimation function, and \(C\) is high-pass filter or soft limiter on solution smoothness. The application of steepest descent algorithms to the solution of the minimization problem (2) results in the iterative scheme

\begin{equation}
\hat{f}^{k+1} = (I - \alpha \beta C^TC)g[Hf^{k}] + \beta H^T[\hat{g} - Hf^{k}]
\end{equation}

where \(\hat{f}^{k+1}\) is the estimation of \(f\) on \(k+1\) iteration, \(\beta\) is a relaxation parameter, \(\text{H}^T\text{H}\) denotes matrix transpose and \(\Psi\) is the projection operator onto the convex set of nonnegative solutions. Operator \(C = I - \alpha \beta C^TC\) represents the low-pass filter that smoothes the restored image to prevent noise fluctuations in the solution and it is a soft constraint on smoothness. Estimate \(\hat{g}\) is the solution of \(\Psi^\alpha(g - \hat{g}) = 0\) and \(\Psi^\lambda(x) = d\Psi^\lambda(x) / dx\) according to equation (3). The choice of function \(\Psi^\lambda\) could be based on a priori knowledge of noise statistics. The best known solutions of this problem are mean value for Gaussian noise, median for impulse noise and \(\alpha\)-trimmed mean (\(\alpha\)-TM) for mixed noise that will be used in our modeling [1].

### 3.2. Robust restoration based on M-estimator of residual

The robust estimation of residual is performed according to the next criterion

\begin{equation}
\min_{f} \mathcal{D}(f, \alpha) = \rho(g - Hf) + \alpha \|Cc\|^2
\end{equation}

where \(\rho(\cdot)\) has the same robust properties as \(\rho_\alpha(\cdot)\) in eq. (3). The corresponding soft limiter or indicator function will be \(\Psi(r) = \frac{d\Psi(r)}{dr}\) where \(r = g - Hf\). In classical restoration approaches it is assumed \(\Psi(r) = r\) (Fig. 1a), which corresponds to the quadratic nature of LSE methods. But in the proposed approach it is suggested to use \(\Psi(r) = \frac{r}{1 + (r / \theta)^\nu}\) (Fig. 1b) to restrict the influence of outliers on the estimation procedure. In the proposed function the parameter \(\nu\) defines the uncertain range in outlier classification. The corresponding iterative algorithms, that minimize (5) for the indicator functions considered above, are

\begin{align}
\hat{f}^{k+1} &= C \Psi^\alpha[f^{k}] + \beta H^T[Hf^{k} - \hat{g}]
\end{align}

for classical LSE approach

\begin{align}
\hat{f}^{k+1} &= C \Psi^\alpha)f^{k}] + \beta H^T[Hf^{k} - \hat{g}\]
\end{align}

for the proposed approach.
4. Results of computer modeling

This section demonstrates the main properties of the proposed robust restoration algorithms in comparison with the conventional regularized algorithm (6). The comparison measure is defined according to signal-to-noise ratio (SNR) between original image $f$, degraded image $g$ and restored image $f^k$

$$SNR = 10 \log_{10} \frac{\|f\|^2}{\|f - \gamma\|^2}$$

where $\gamma = g$ for the direct observation and $\gamma = f^k$ for the restored image. The 256×256 test radar image received by Spaceborne Imaging Radar-C is depicted in Fig. 2a. This image was blurred by an antenna array with uniform amplitude distribution and errors in phase during reception (i.e. defocused) [2] and then this image was contaminated by additive Gaussian noise with SNR 35dB and impulse noise with 10% of pixels (Fig. 1b) and 20% (Fig. 1c). The overall SNRs are shown in Table I. The images restored by means of the methods described by eqs. (4), (6) and (7) are depicted in Fig. 2 and corresponding enhancement of SNR is shown in Table I.

TABLE I.  
Comparison based on SNR between various methods in robust estimation schemes.

<table>
<thead>
<tr>
<th>Percentage of impulse noise</th>
<th>Direct observation</th>
<th>Tikhonov restoration (6)</th>
<th>Robust residual estimation (7)</th>
<th>Robust prefiltering (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>6.9dB</td>
<td>6.5dB</td>
<td>16.9dB</td>
<td>18.2dB</td>
</tr>
<tr>
<td>20%</td>
<td>4.5dB</td>
<td>3.9dB</td>
<td>14.5dB</td>
<td>17.4dB</td>
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</tbody>
</table>

5. Conclusions

The comparative analysis of robust restoration algorithms applied to radar image indicates that robust prefiltering provides better image restoration than robust residual estimation. Both of the proposed approaches make it possible to restore images with much higher SNR in comparison with conventional Tikhonov regularization. Further investigation will be directed towards the development of a generalized adaptive robust approach to the definition of composite functions in robust regularized criterion.
Fig. 2. The results of computer modeling: (a) original image; (b) image degraded by phase errors in imaging antenna aperture with additive Gaussian noise with SNR 35dB and 10% impulse noise and (c) 20% impulse noise; (d) and (g) the estimate of Tikhonov regularized (6) approach ε=0.01, (e) and (h) robust residual estimates (7), (f) and (i) restorations with robust prefiltering (4) based on ε-TM (3x3 window) for images (b) and (c) respectively.

References
