ROBUST IMAGE RESTORATION BASED ON M-ESTIMATION CONCEPT AND PARAMETRIC MODEL OF IMAGE SPECTRUM

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Abstract: *The paper presents robust approach to image restoration problem that combines the properties of classical regularized iterative algorithms and robust features based on the concept of M-estimators. The proposed approach makes possible to suppress the influence of impulse noise on restoration process. To enhance the convergence rate of iterations, remove ringing effect and preserve edges and fine details in the restored image the parametric constrain on amplitude image spectrum is used that bounds from below the energy of the restored image at high frequencies opposite regularization. The robustness of the proposed algorithm is demonstrated.*

Key words: *image restoration, robust estimation, regularization, iterative algorithm.*

1. INTRODUCTION

In a lot of applications the observed image is degraded by spatial blurring of imaging system and corrupted by noise of different nature. Spatial image blurring is conditioned by the different factors which depend on the particularities of observation. The main blurring factors, that lead to the ambiguity of measurement, are the finite imaging aperture, spatial atmosphere turbulence, motion blur, out-of-focus observation, sparseness of aperture that conditions the high level of side lobes in point spread function and nonoptimal beamforming caused by phase errors in imaging aperture. Additionally, situation is complicated by errors of measurement or noise of receiver and registration equipment that could be modeled by additive Gaussian noise. From the other side, additional problem appears which is caused by ununiform sampling owing to the failures in spatial scanning system or the physical nature of possible spatial sampling that condition the information loss. The last two distortions could be presented by "salt-and-pepper" model of impulse noise.

Most existing restoration techniques have a common unified structure that is expressed by regularization strategy. However, conventional Tikhonov regularization is characterized by global image smoothing and low efficiency in the case of impulse noise presence. To overcome these disadvantages M-estimation concept is used [1] in combination with the lower bound constrain of image spectrum expressed by parametric image spectrum model.

2. PROBLEM FORMULATION

In the general case of image degradation caused by the above mentioned blurring factors and image distortion conditioned by Gaussian and impulse (mixed) noise the next model is used

$$g = \begin{cases} Hf + n, \text{ with probability } 1 - p \\ n_{imp}, \text{ with probability } p \end{cases} \quad \textbf{(1)}$$

where g represents degraded image, s is signal component in measurement and n represents the corresponded Gaussian noise component. H represents the linear spatially invariant blurring operator and f denotes original image, n_{imp} is impulse noise which occurs with probability p and

$$n_{imp} = \begin{cases} g_{\min}, \text{ with probability } 1 - q \\ g_{\max}, \text{ with probability } q \end{cases} \quad \textbf{(2)}$$

where $g_{\min}$ and $g_{\max}$ are minimum and maximum values of the dynamic image range and q denotes the corresponded occurrence probability. In our modeling we suppose q=0.5.

3. PROBLEM SOLUTION

The solution of restoration problem consists in the estimation or retrieval of original image on the base of degraded image using available *a priori* information about model of observation (1) and knowledge of blurring function and statistics of noise and original image given in the form of image smoothness, solution constrains on some specific image features and parametric models. This refers to the solution of inverse problems which are known to be ill-posed ones. Among the large number of linear and nonlinear methods developed for the solution of the estimation problems we have chosen here the approach based on the theory of M-estimators due to its robust features [2].

3.1. Robust restoration based on M-estimation

The robust estimation of residual based on M-estimation concept is performed according to the next criterion

$$\min_{\hat{f}} \Phi[\hat{f}] = \rho(g - H\hat{f}) + \alpha\|C\hat{f}\|^2 \quad \textbf{(3)}$$

where $\|.\|$ is matrix norm, α is regularization parameter, $\rho(.)$ is robust objective estimation function, C is high-pass filter. The application of steepest descent algorithms to solution of minimization problem (3) results in the next iterative scheme

$$\hat{f}^{k+1} = \hat{f}^k - \beta\, d\Phi[\hat{f}^k] / d\hat{f}^k \quad \textbf{(4)}$$

where $\hat{f}^{k+1}$ is the estimation of f on k+1 iteration, β is a relaxation parameter, and

$$\frac{d\Phi[\hat{f}]}{d\hat{f}} = -2H^T\Psi(g - H\hat{f}) + 2\alpha C^T C\hat{f} \quad \textbf{(5)}$$

where $\Psi(r) = d\rho(r)/dr$ is indicator function, $r = g - H\hat{f}$, "T" denotes matrix transpose. In classical restoration approaches it is assumed $\Psi(r) = r$ (Fig.1 dotted line) that corresponds to the quadratic nature of LSE method and in the proposed approach it is suggested to use $\Psi(r) = r / (1 + (r/\theta)^{2\upsilon})$ (Fig. 1 solid curve) to restrict the influence of outliers.

The resulted iterative algorithms, that minimizes (3) for the considered above indicator function, is

$$\hat{f}^{k+1} = C_S \hat{f}^k + \beta H^T \Psi(g - H\hat{f}^k) \quad \textbf{(6)}$$

The generalized operator $C_S = I - \alpha\beta C^T C$ represents the low-pass filter that smoothes restored image to prevent noise fluctuations in the solution and it is soft constrain on smoothness.

3.2. Robust restoration with hard constrains on solution

The use of hard constrains on the solution leads to the iterative algorithm

$$\hat{f}^{k+1} = C_S \Re[\hat{f}^k] + \beta H^T \Psi(g - H\Re[\hat{f}^k]) \quad \textbf{(7)}$$

where $\Re$ is the projection operator onto convex set of nonnegative solutions and solutions with the given properties of amplitude image spectrum:

$$\Re = C_N C_M \quad \textbf{(8)}$$

where C_N is constrain on nonnegativity and C_M is constrain on *a priori* given module of solution. Unfortunately, constrain C_M is not directly available in many practical applications. Therefore, parametric model in frequency domain is used that takes into account the spatial anisotropy of real image spectra. The upper bound of the restored image module is determined by regularization and the above constrain enables to determine the lower bound using next constrain

$$C_M[\hat{f}^k] = \begin{cases} \Im^{-1}\{|\hat{F}_L(\omega)|, \varphi_{\hat{f}^k}(\omega)\}, \; if \; |H(\omega)| \le T_Z, \\ \hat{f}^k, \; if \; |H(\omega)| > T_Z, \end{cases} \quad \textbf{(9)}$$

where $|\hat{F}_L(\omega)| = (|\hat{F}_{pm}(\omega)| + |\hat{F}^k(\omega)|)/2$, $T_z = 0.1\max|H(\omega)|^2 = 0.1|H(0)|^2$ and $|\hat{F}_{pm}(\omega)|$ is the estimation of mean amplitude spectrum of initial image according to the parametric model considered below, $\varphi_{\hat{f}^k}(\omega)$ is phase of the restored image f^k, $|H(\omega)|$ is amplitude spectrum of blurring function, and $\Im^{-1}\{.\}$denotes inverse FFT. In our approach the above constrain is applied only for $k = 0$, i.e. in eq.(9) $\varphi_{\hat{f}^k} = \varphi_g$ and $|\hat{F}^0(\omega)| = |\hat{G}(\omega)|$ where $\hat{G}(\omega) = \Im\{med_L(g)\}$ is spectrum of median pre-filtered image with window size *L*.

To estimate the mean spectrum the parametric model was used. The model is based on the fact that most spectra of real images have the exponential decay character. The model of the sum of 3 exponential decays was used to fit the module of image spectrum. According to the proposed method the estimation of the mean spectrum is modeled by [3]

$$|\hat{F}_{pm}(\omega)| = \sum_{p=1}^{3} a_p \exp(-\omega / \gamma_p) \quad \textbf{(10)}$$

where $\{a_p\}$ and $\{\gamma_p\}$ are parameters of the model which are found on the base of the functional minimization:

$$\Phi[a, \gamma] = \|\hat{F}'(\omega) - \hat{F}_{pm}(\omega)\| + \lambda C[a, \gamma] \quad \textbf{(11)}$$

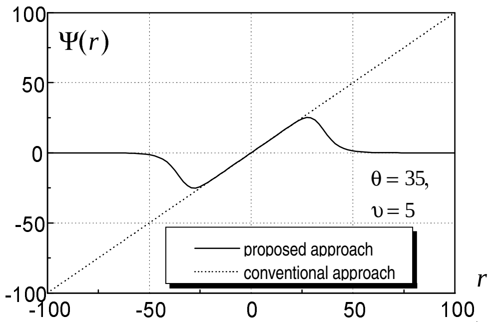

Figure 1. Indicator functions $\Psi(r) = r$ (dotted curve) and $\Psi(r) = r \Big/ \left(1 + (r/\theta)^{2\upsilon}\right)$ (solid curve).

where λ is Lagrangian multiplier, $C[a, \gamma]$ is constrain on the possible ranges of the values $\{a_p\}$ and $\{\gamma_p\}$ obtained from the averaged spectrum of the group of images, $\hat{F}'(\omega)$ is the estimation of the initial image spectrum in the range of cut-off frequency $\omega_{cut-off}$ of the imaging system taken on the level of 0.1 from maximum $|H(\omega)|^2$

$$\hat{F}'(\omega) = \hat{G}(\omega) \,/\, |H(\omega)|^2, \text{ if } |\omega| \le \omega_{cut-off}. \qquad \textbf{(12)}$$

4. RESULTS OF COMPUTER MODELING

This section demonstrates the main properties of the proposed robust restoration algorithms. The comparison measure is defined according to SNR between original image *f*, degraded image *g* and restored image f^k

$$SNR = 10\log_{10}\frac{1}{\delta_{\hat{f}}^2}, \; dB, \qquad \text{where } \delta_{\hat{f}} = \frac{\|f - \gamma\|}{\|f\|} \qquad \textbf{(13)}$$

where $\gamma = g$ for the direct observation and $\gamma = f^k$ for the restored image.

The numerous experimental restoration made possible to establish the optimal parameters of the proposed indicator function $\theta = 35, \upsilon = 5$, i.e. that solves the compromise between impulse noise suppression and image degradation caused by the decrease of the residual dynamic range.

The 256x256 test image "Cameraman" was chosen (Fig. 4a). This image was blurred (Fig. 4b) by law-pass filter with amplitude spectrum shown in Fig. 4c. Additive zero-mean Gaussian noise was added to the blurred image to obtain high enough SNR 50 dB to investigate the properties of the proposed algorithm concerning impulse noise. The resulted image was corrupted by salt-and-pepper impulse noise with 20% (Fig. 4d), 50% (Fig. 4e) and 70% (Fig. 4f) of all pixels. The restored images by means of the proposed method are depicted in Fig. 4 (g-i). Spectrum $|\hat{G}(\omega)|$ was estimated on the base of median pre-filtering with window size chosen according to the condition of the complete removal of impulse component: 10% - 3; 20-30% - 5; 40% - 7; 50% - 9; 60% - 11; 70% - 13.

The performed analysis is demonstrated in details on the example of 20% impulse noise. The convergence rate comparison with respect to Fig. 4d is shown in Fig. 2. The figure demonstrates the essential reduction of error for the proposed method with module estimation on the base of median pre-filtered image and its robust features in comparison with the *a priori* given module from original image. **Table I** summarizes the obtained results according to criterion (14) for 30 iterations. Comparing the last two rows we can conclude that the proposed approach provides about optimal image restoration using pre-filtered image and superior performance in comparison with the rest of methods.

Consequently, the robust features of the proposed algorithm were investigated with respect to percentage of impulse noise that is depicted in Fig. 3 in comparison with SNR for the direct observation and median filtering.

Table I. Comparison between various methods in robust estimation schemes based on SNR for 20% and 50% impulse noise.

Method	SNR, dB for 20%	SNR, dB for 50%
Direct observation	6.22	2.44
Tikhonov regularization	4.50	-1.21
Without constrain on module	20.91	16.87
Module from noisy image	21.41	17.42
Module from pre-filtered image	24.04	20.30
Module from original image	24.14	20.35

5. CONCLUSION

The robust approach to image restoration based on the concept of M-estimators is proposed. The new influence function is considered and corresponded algorithm is developed. To enhance the iteration convergence and suppress the ringing effect the parametric constrain on image module is used.

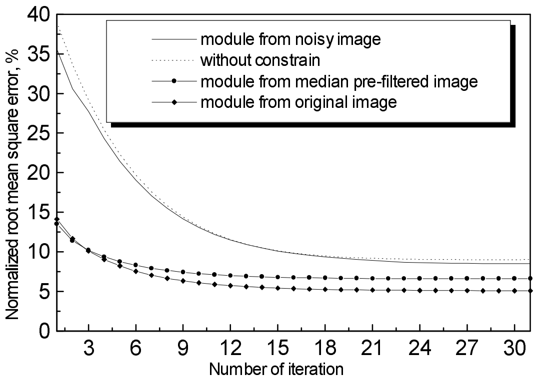

Figure 2. Convergence rate comparison: error reduction with respect to norm (13).

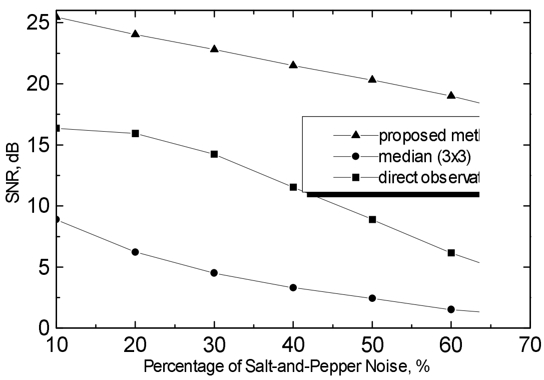

Figure 3. Performance evaluation of the proposed algorithm.

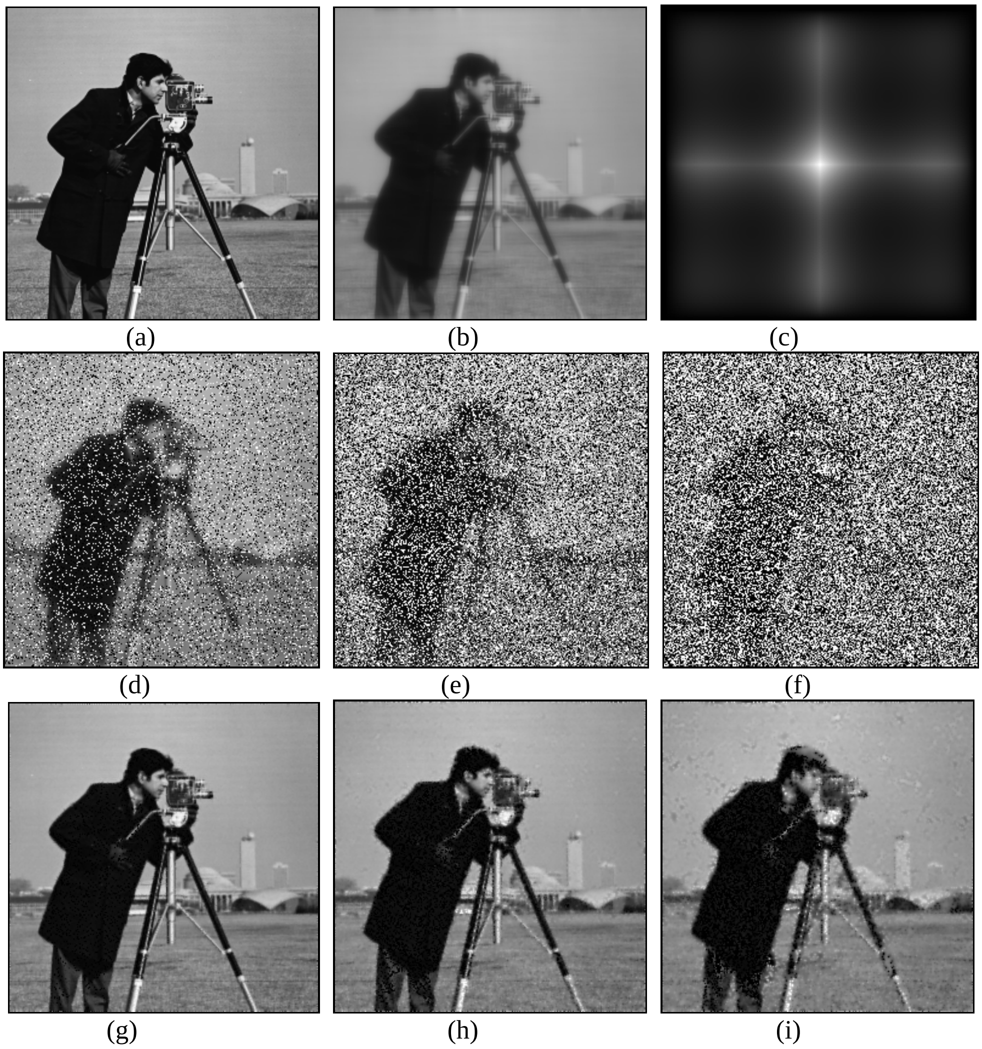

Figure 4. Restoration of test image "Cameraman": (a) Original image 256x256x8bit; (b) blurred image by low-pass filter with amplitude spectrum (c); image (b) corrupted by 20% (d), 50% (e) and 70% (f) salt-and-pepper impulse noise. (g-i) results of restoration on the base of the proposed approach with $\alpha = 0.01$ for corresponded images (d-f).

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