Paper

Generalized radar/radiometry imaging problems

Ivan Prudyus, Sviatoslav Voloshynovskiy, Andriy Synyavskyy,
Taras Holotyak, and Leonid Lazko

Abstract — In the paper the results of spatio-temporal imaging simulation based on radar, synthetic aperture radar (SAR) and radiometry systems are presented. The analytical relationship between object scattering/emitting and the formed image is given and the general approach for the description of imaging system by means of Frendholm equation solution is developed. The potential limit of image resolution based on Rao-Cramer inequality is estimated.

Keywords* — *radiometry systems, synthetic aperture radar, spatio-temporal imaging.

1. Introduction

Resolution of radar and radiometry imaging systems is always constrained because of their finite spatio-temporal bandwidth that is determined by the constructive and technological particularities. That causes the decrease of received image quality. Advances in modern image and signal processing techniques open the new possibilities of real time image processing. Thus, the problem of adequate simulation of image formation for the development of optimal algorithms of resolution increasing and obtained image quality improving is very actual.

In this paper three main classes of imaging systems (monostatic radar, SAR and radiometer) are considered [1]. The distortions caused by imaging systems are determined as the analytical relation between scattering/emitting object's ability and the obtained image. Scattering $Q(x,y)$ is a local object characteristic that is described by relation between incident and reflected waves. Emitting $Q_p(x,y)$ property is defined by radiating particularities of investigated object.

2. Radar image formation

In radar imaging systems the range portrait is formed based on the delayed scattered signal from the different object's parts [2, 3]. This enable to obtain one-dimensional object presentation during one radiation period. In this case, the delay determinates coordinates of scattering area, signal magnitude permits to estimate scattering coefficient and antenna directional pattern has to satisfy the condition of uniform object radiation and reflected-signal receiving (Fig. 1). This principle is widely used for two-dimensional radar imaging $[2-6]$, where resolution in second coordinate is satisfied by scanning with narrow antenna directional pattern. Developed model of radar imaging system was created under assumption about flat surface of investigated objects, isotropic scattering property of the object's elements, and absence of secondary reflection:

$$s_{echo}(t)=\int_{0}^{T_{rad}} Q\left[\sqrt{c^2(t-\tau)^2-H^2}\right] s_{rad}(\tau)\cdot\frac{c^2(t-\tau)\,d\tau}{\sqrt{c^2(t-\tau)^2-H^2}}\,, \qquad (1)$$

where $s_{rad}(t)$ is the radiated signal; T_{rad} denotes the radiated signal duration; c is the wave propagation velocity; H is the altitude of radar position.

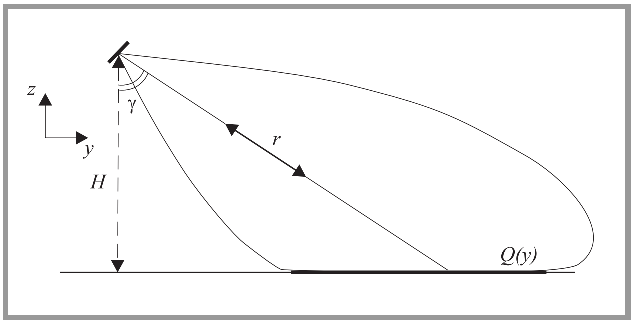

Fig. 1. The geometry of radiation at the range portrait formation by radar system.

Temporal function of received signal $s_{echo}(t)$ transformed into corresponding spatial coordinate system is a range portrait $I(Y)$ and Eq. (1) can be rewritten as:

$$I(Y)=\int_{y_{\min}}^{y_{\max}} Q(y)\,G_d[y,Y]\,dy\,, \qquad (2)$$

where $G_d[y,Y]$ denotes the transform kernel Eq. (2) or system function determined by radiated signal; $(y_{\max}, y_{\min})$ is interval of scattering property investigating $Q(y)\,G_d[y,Y]$ describes the temporal distortion features. In the case of $G_d(t,\tau)=s_{rad}(t-\tau)$, Eq. (2) will be presented by a convolution, and distortion will be invariant in respect to the range coordinate.

3. Radar imaging system with synthetic aperture

To satisfy high quality of remote sensing, multiposition radar systems with coherent processing, i.e. SAR are widely used, that permits to significantly improve cross-range resolution [4, 5]. Oppositely to ordinary radar, the formation of synthesized directional pattern is performed by corresponding spatio-temporal processing (Fig. 2). In each discrete position with step Δx the signal $s_{rad}(t)$ is radiated and echo-signal is received. Taking into account spatial filtering properties of the transmitting and receiving antennas, echo-signal $s_{rec}(t)$ can be presented according to Eq. (1). One of the particularities of these systems is Doppler frequency shift effect that depends on the wave propagation direction.

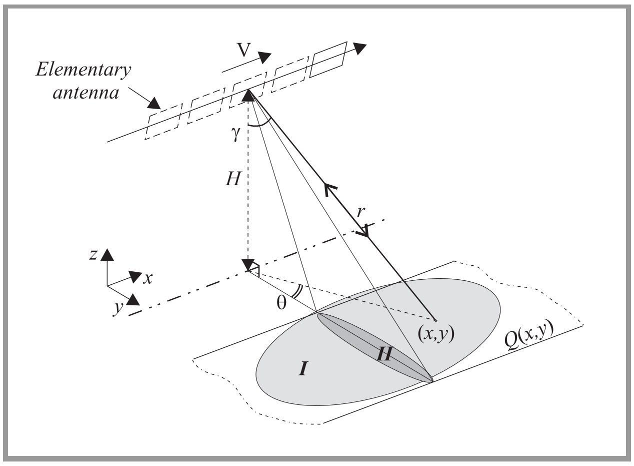

Fig. 2. Aperture synthesizing in radar imaging systems. Explanations: *I* – elementary antenna pattern, *II* – synthetic aperture pattern.

Synthesis process consists in the coherent summing of the received signals

$$U^i(y) = \sum_{k=-L}^{L} C_k s_{rec}^{i+k} \left(\frac{2\sqrt{y^2+H^2}}{c} \right), \qquad (3)$$

where $U^i(y)$ is the complex discrete-continuous SAR image. Complex coefficients C_j are equivalent to the current distributions in the synthetic aperture. Depending on the choice of C_j Eq. (3) gives a possibility to obtain different synthetic aperture directional patterns, to select the orientation direction of directional antenna pattern main lobe, to focus the synthetic aperture on the certain range. Model (3) does not permit to obtain the necessary resolution in the range coordinate and compensate the Doppler frequency shift that causes resolution decreasing.

The developed model was created under assumption of discrete system carrier moving with synthesizing step Δd. Taking into account Eqs. (1), (2) and (3) SAR model can be expressed as:

$$I(X,Y) = \int_{v_{\min}}^{v_{\max}} \int_{y_{\min}}^{y_{\max}} Q\Big((v-X),y\Big)\, G_{sar}[v,y,Y]\, dy\, dv, \qquad (4)$$

where transformation kernel $G_{sar}[v,y,Y]$ is being defined SAR carrier altitude H, its velocity V, synthesizing step Δd, as well as radar parameters: waveform of radiated signals $s_{rad}(t)$, its carrier frequency ω_0, complex coefficients of coherent processing C_k, directional properties of transmitting $F_{tran}(\gamma,\theta)$ and receiving $F_{rec}(\gamma,\theta)$ antenna directional patterns, and described by the following equation:

$$G_{sar}[v,y,Y] = c^2 \cdot \sum_{k=-M}^{M} C_k F_{tran}[\gamma,\theta] \cdot F_{rec}[\gamma,\theta] \times$$
$$\times \frac{D^2+H^2}{T} \cdot s_{rad}\left(\frac{2Y}{c} - \frac{\sqrt{D^2+H^2}}{c}\right) \exp\left\{ \frac{-j2\omega_0\sqrt{D^2-y^2}\,V}{c^2\sqrt{D^2+H^2}} \times \right.$$
$$\left. \times \left(2Y - \sqrt{D^2+H^2}\right) \right\}, \qquad (5)$$

where $\theta = \left(\operatorname{arctg}\left(\frac{\sqrt{D^2-y^2}}{y}\right)\right)$, $\gamma = \left(\arccos\left(\frac{H}{\sqrt{D^2+H^2}}\right)\right)$ and $D = \sqrt{(v+k\Delta d)^2+y^2}$.

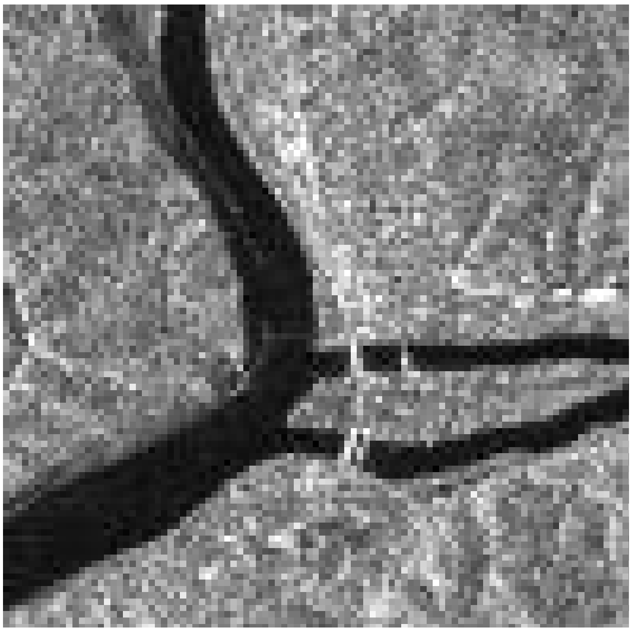

Fig. 3. Tested scattering ability $Q(x,y)$.

In alternative to SAR models based on Radom transform [6] model (4) does not need the coordinate transformation, that permits to simplify the processing algorithms. To show the particularities of the above mentioned model the test image presented in Fig. 3 was chosen. The simulation was performed for Gaussian radiated signal and uniform field distribution in the elementary transmitting and receiving antennas and different kinds of coherent signal processing methods (Fig. 4). Obtained results confirm possibility to form the narrow directional pattern of synthesized aperture by means of coherent summation with quadratic phase and time delay compensation. Kernel (5) shows that SAR distorting impact is invariant to x, and non-invariant to y that is explained by different nature of image formation in these coordinates.

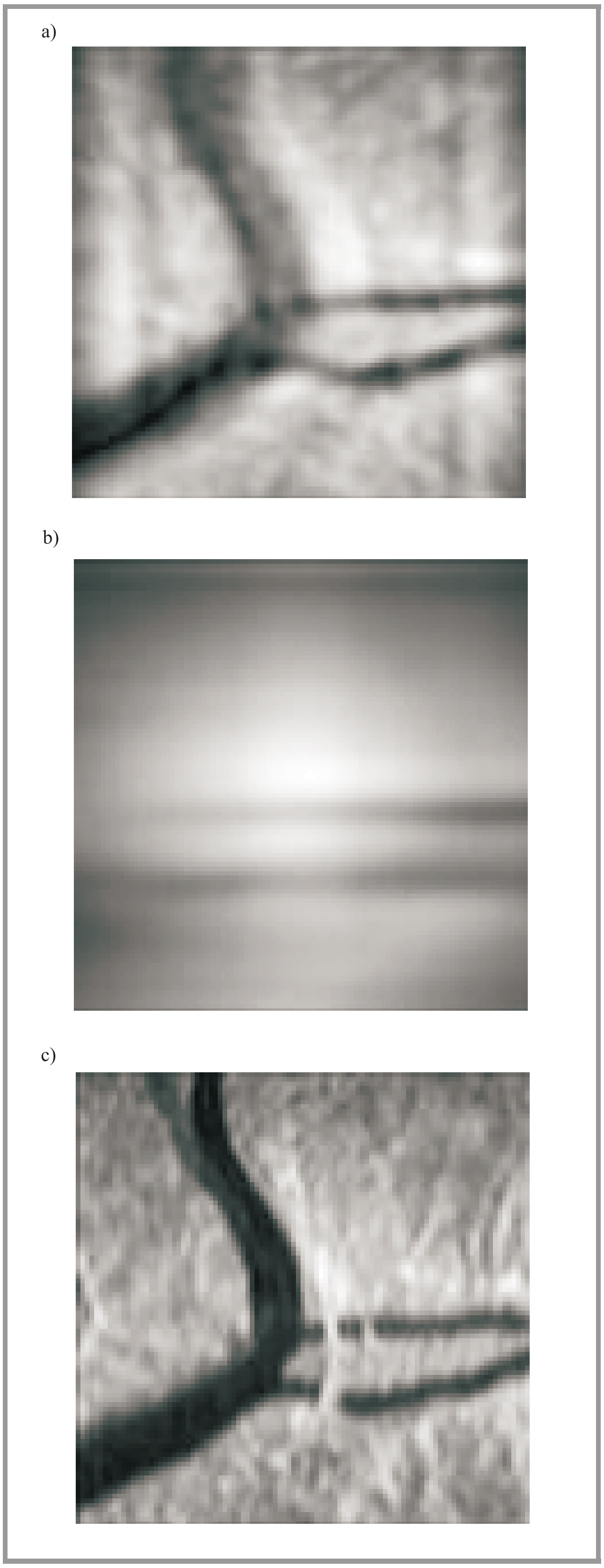

Fig. 4. Obtained SAR image $I(X,Y)$ with 5 elements (a), with 100 elements and without quadratic phase delay compensation (b), with 100 elements and corresponding quadratic phase delay compensation (c).

4. Radiometry imaging systems

In radiometry systems image formation process is being performed by two-dimensional scanning [7, 8]. Radiometry receiver is energy device, thus, the received image characterizes energetic properties of the studied object emission. This class of systems can be described by following model:

$$I(X,Y) = \int_{\vartheta_{\min}}^{\vartheta_{\max}} \int_{\eta_{\min}}^{\eta_{\max}} Q_p(\eta,\vartheta)\cdot G_{pasiv}[X-\eta, Y-\vartheta]\, d\eta\, d\vartheta\,, \tag{6}$$

where $(\eta_{\max}, \eta_{\min}, \vartheta_{\max}, \vartheta_{\min})$ denotes the scanning region; $G_{pasiv}[X-\eta, Y-\vartheta]$ is transformation kernel and depends on field distribution in aperture $e(x,y)$:

$$G_{pasiv}[\eta,\vartheta] = \left|\Im\left\{e(x,y)\right\}\right|^2, \tag{7}$$

where $\Im\{e(x,y)\}$ denotes two-dimensional Fourier transform.

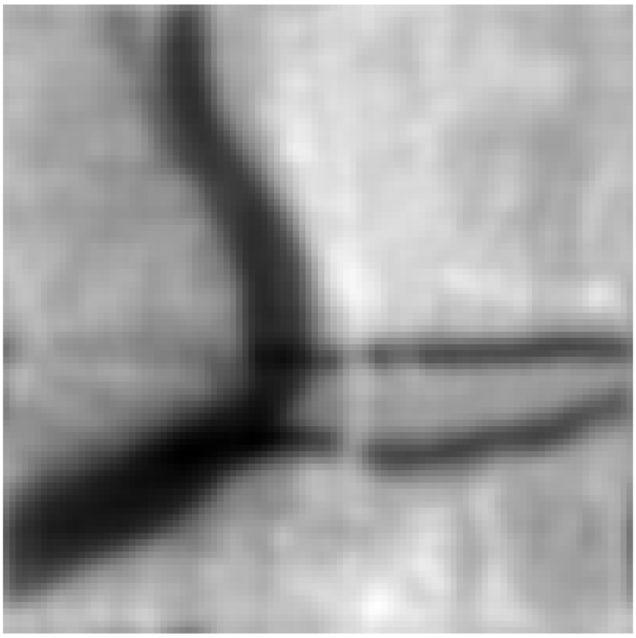

Fig. 5. Radiometry image $I(X,Y)$ obtained on the base of the model (7).

Image formed by radiometry systems according to Eq. (6) is presented in Fig. 5. The distortions caused by radiometry imaging systems have spatially invariant character and are determined by the form of a system function. The significant losses in high spatial frequency band cause typical blurring of the obtained image.

5. Generalized linear imaging model

The presented above models of the image formation systems Eqs. (2), (4) and (6) are deterministic ones. However, the real systems usually are under impact of many stochastic factors, such as received noise, error of imaging system geometrical parameters estimation, system parameters errors (radiated signal and direction pattern forms). The complicated stochastic nature of these parameters do not allow to obtain precise statistic model of radar/radiometry imaging systems. Taking into account large number of stochastic parameters and the linearity of image formation models the additive Gaussian component can be assumed due to central limit theorem assumption. In this case the generalized model of radar/radiometry imaging systems is described by the following expression:

$$I(X,Y) = \int_{x_{\min}}^{x_{\max}} \int_{y_{\min}}^{y_{\max}} Q(x,y) \cdot G[x,X,y,Y]\, dy dx + n(X,Y), \tag{8}$$

where $G[x,X,y,Y]$ is the point spread function, and n is the additive Gaussian component.

Based on the presented models, the synthesis of image processing algorithms is possible and quality improving can be formulated based on the inverse solution of first kind Frendholm equation. Correctness and robustness issues of this problem solution will be determined by kernel form [9], type of systems and their parameters that have also impact on the noise statistics.

This approach gives possibility to reduce blurring impact of point spread function and remove the noise component. The synthesized based on this approach SAR image processing algorithms permit automatically to compensate the quadratic phase distortions and moving of SAR carrier.

6. Potential limit of restoration accuracy. Restoration methods

A lot of methods of Frendholm integral equation solution are known [9 – 14]. Deterministic methods based on prior information are given in [3]. Selection of solution methods is being performed according to the type of integral equation kernel or structure and size of equation system matrix. Singular operators are characterized by nonstability of solution to calculation errors, imprecission of initial data or stochastic component existing in right part of equation.

Radar and radiometry image restoration (8) are often performed under significant noise impact. Stochastic character of the radar image formation shows necessity to use stochastic methods of problem (8) solution. These methods give possibility not only to find stochastically correct solution, but also to estimate potential solution limit.

Problem (8) can be presented by system of linear equations:

$$X = GA + n, \tag{9}$$

where G is $M \times M$ matrix with $g_{i,j}$ elements; X and A are obtained and original image respectively; n is random vector with Gaussian distribution $N(0,\sigma_n^2)$. Each element x_i of vector X can be presented by linear combination of unknown parameters a_j. In the case of prior information about distribution law of a_j absence, Eq. (9) can be solved by maximum likelihood (ML) principle [15]. Element x_i has the Gaussian distribution with mean $\sum_{j=1}^{M} g_{i,j} \cdot a_j$ and variance σ_n^2. Then taking into account statistical independence of x_i, likelihood function can be written as:

$$\begin{aligned} p(X|A) &= \prod_{i=1}^{M} p(x_i \mid A) = \\ &= \frac{1}{\left(\sqrt{2\pi\sigma_n^2}\right)^M} \exp\left\{ -\sum_{i=1}^{M} \left[\left(x_i - \sum_{j=1}^{M} g_{i,j} a_j\right)^* \times \right.\right. \\ &\left.\left. \times \left(2\sigma_n^2\right)^{-1} \left(x_i - \sum_{j=1}^{M} g_{i,j} a_j\right) \right] \right\}. \end{aligned} \tag{10}$$

Solving Eq. (9) by means of ML function corresponds to solving of equations' system $G^T X = G^T G A$ that coincides with the least squares approach [9]. Potential accuracy of Eq. (9) solution can be found from Rao-Cramer inequality [15]. From Eqs. (10) and (11) the accuracy of unknown parameter a_j based on ML principle is calculated as its estimation variance:

$$\begin{aligned} E\left[\hat{a}_{i_{ml}} - a_i\right]^2 &\geq \left(-E\left[\frac{\partial^2 \ln\left(p(X \mid A)\right)}{\partial a_i \partial a_i} \right] \right)^{-1} = \\ &= \sigma_n^2 \left(\sum_{l=1}^{M} |g_{l,i}|^2 \right)^{-1}, \qquad i = \overline{1,M}. \end{aligned} \tag{11}$$

The found accuracy is the same for all unknown parameters and defined by the relation between random component variance and sum of squared column's elements of matrix G. Inequality (11) gives possibility to estimate potential accuracy of the radar/radiometry image restoration. Precision of the restoration without prior information is equal to the relation of noise variance to the squared norm of point spread function. In the case of spatially variant linear operators, e.g. Eqs. (2) or (4), point spread function changes its form in dependence on the image coordinates, therefore, accuracy will be defined by different relationship.

7. Conclusions

The results of radar and radiometry imaging systems analysis show possibility to describe these system by the linear model. The analytical form of operator transforms for efficient image formation simulation is determined for the classical types of these systems. Blurring of radar and radiometry images is caused by the finite spatio-temporal bandwidth. Nonoptimality of the coherent processing in SAR also decreases image quality. Based on the developed models the approach to radar and radiometry image quality improving that consists in the image processing by means of restoration methods synthesized by the mentioned models is generalized. The potential limit of image restoration accuracy under Gaussian noise impact is determined.

Variance of image estimation is proportional to the noise energy and inversely proportional to the squared norm of point spread function, that permits to make some recommendations about wave form type and antenna parameters. According to the developed model of radar range portrait formation the point spread function corresponds to radiated signal form that points out on the possibility of signal with large base usage [16]. This kind of signals has high energy because of large duration and wide band because of the complicated modulation. Spatial properties of the imaging systems are defined by antenna parameters and directly depended on the aperture magnitude-phase distribution. Thus, usage of complicated distributions permits to satisfy the robustness of image restoration algorithms to noise level and to obtain high accuracy.

References

[1] M. I. Skolnik, *Spravočnik po Radiolokacii*. Vol. 4. Sov. Radio, 1978 (transl. Engl).

[2] V. Guglielmi, F. Castanie, and P. Piau, “Application de methodes super-resolvantes au traitement des donnees d’un radar a synthese d’ouverture”, in *Proc. 5 Int. Conf. GRETSI’95*, Juan-Les-Pins, France, 1995, pp. 1097–1100.

[3] Je. I. Klepfer, I. N. Prudyus, A. T. Synyavskyy, and L. V. Lazko, “Vikoristannja nelinijnich obmežen’ v iteracijnich metodach vidnovlennja radiolokacijnich signaliv”, *Radiotechnika ta Telekomunikacii: Visnik DU “LP”*, no. 387, pp. 36–45, 2000.

[4] I. N. Prudyus, A. T. Synyavskyy, and V. P. Ostap, “Simulation of spatial-temporal signal processing in imaging systems with synthetic aperture”, in *Proc. 3rd Int. Conf. Anten. Theory Techn. (MKTTA’99)*, Sevastopol, Ukraine, 1999, pp. 233–236.

[5] I. N. Prudyus, A. T. Synyavskyy, and V. P. Ostap, “Modeljuvannja procesu formuvannja radiolokacijnich zobražen’ v sistemach z sintezovanoju aperturoju”, in *Proc. Int. Conf. MME’99*, Lvov, Ukraine, 1999, pp. 221–222.

[6] M. Lindberg, “Radar image processing with the radon transform”. Technical reports. European Consortium for Mathematics in Industry. Programme in applied mathematics, no. 5, 1997.

[7] I. N. Prudyus, Z. D. Grytskiv, and S. V. Voloshynovskiy, “Radiometry image processing based on nonlinear iterative methods”, in *Proc. Workshop Des. Methodol. Signal Proc.*, Zakopane, Poland, Aug. 1996, pp. 21–27.

[8] I. N. Prudyus, S. V. Voloshynovskiy, and T. S. Holotyak, “Investigation of spatial antenna system characteristics in active and passive imaging systems”, in *Proc. Int. Conf. (TCSET’2000)*, Lvov-Slavsko, Ukraine, 2000, pp. 124–125.

[9] A. N. Tichonov and V. Ja. Arsenin, *Metody Rešenija Nekorrektnych Zadač*. Nauka, 1979.

[10] G. I. Vasilenko and A. M. Tatarinov, *Vosstanovlenie Izobraženij*. Radio i Swjaz’, 1986.

[11] M. Kilmer and D. P. O’Leary, “Choosing regularization parameters in iterative methods for ill-posed problems”. Technical reports CCR 95-03126, CCR 97-32022, University of Maryland, Dept. of Computer Science, College Park, MA, 1997.

[12] J.-L. Starck, F. Murtagh, and A. Bijaoui, “Multiresolution support applied to image filtering and restoration”, *Comp. Vis., Graph. Image Proc.*, no. 34, pp. 234–250, 1996.

[13] M. Nielsen, L. Florack, and R. Deriche, “Regularization and scale space”. Rapport de recherche, Institut national de recherche en informatique et en automatique; no. 2352, programe 4 (Robotique, image et vision), Sophia-Antipolis, France, 1994.

[14] J. G. Nagy and D. P. O’Leary, “Fast iterative image restoration with a spatially-varying PSF”, *SIAM J. Sci. Comput.*, no. 2, pp. 474–489, 1997.

[15] G. Van Tris, *Teorija Obnaruženija, Ocenok i Modulacii*. Sov. Radio, 1972.

[16] I. N. Prudyus, M. M. Sumyk, and A. T. Synyavskyy, “Stability of matched filtering of polyphase signals to Doppler shift”, in *Proc. Reg. Conf. Milit. Commun. Inforam. Syst. (RCMCIS’99)*, Zegrze, Poland, Oct. 1999, pp. 104–109.

Ivan Prudyus received the Ph.D. degree in radio engineering from Kaunass Polytechnic Institute in 1980. From 1981 he has worked as Associate Professor at Radio Devices Department of Radio Engineering Faculty, Lvov Polytechnic Institute, and from 1991 he is a Dean of Radio Engineering Faculty, Lvov Polytechnic National University, Lvov, Ukraine. His research interests include antenna and VHF technique, signal processing, systems of recognition and control, receive information systems. He is Lecturer of theory of antennas and VHF devices, telecommunication devices, electromagnetic compatibility.

iprudyus@polynet.lviv.ua
Radio Engineering Faculty
Lvov Polytechnic National University
S. Bandery st 12
Lvov, 79013, Ukraine

Sviatoslav Voloshynovskiy received the Ph.D. degree in radio engineering from Lvov Polytechnical National University in 1996. From 2000 he is an Associated Professor of Department of Radio Electronic Devices and Systems at Lvov Polytechnic National University, Lvov. Also, he is an Assistant Professor of Computer Science Department, University of Geneva, Switzerland. His research interests include radar and radiometry imaging system, antenna design, adaptive image restoration, image enhancement, image compression, and inverse problems.

svolos@cui.unige.ch
CUI, University of Geneva
24 Genral Dufour
4 Geneva 1211, Switzerland

Andriy Synyavskyy was born in Lvov, Ukraine in 1974. He received the radio-engineering diploma and M.Sc. degree from Lvov Polytechnic National University in 1996 and 1997, respectively. After postgraduate courses he joined the Lvov Polytechnic National University on Radio Engineering Faculty in 2000, where is currently working as Junior Scientist above the research projects in the areas of signal/image processing and toward the Ph.D. degree in television- and radio-engineering. His research interests include the description, modeling and characterization of high-resolution radar imaging processes, deterministic and stochastic investigation of microwave scattering as well as creation of radar signals processing methods.

sinat@polynet.lviv.ua
Lvov Polytechnic National University
S. Bandery st 12
Lvov, 79013, Ukraine

Taras Holotyak received the M.Sc. and Ph.D. degrees from Lvov Polytechnic National University in 1997 and 2001, respectively, all in radio engineering. In 2001 he joined the Department of Radio Electronic Devices and Systems at Lvov Polytechnic National University, Lvov, Ukraine, where he is now an Assistant Professor. His research interests are in remote sensing, digital image processing, and computer vision, focusing on radiometry image formation, nonlinear adaptive image restoration, and human visual systems models.
taras@polynet.lviv.ua
Lvov Polytechnic National University
S. Bandery st 12
Lvov, 79013, Ukraine

Leonid Lazko was born in 1975 in Kremenets. From 1992 till 1997 he studied in Lvov Politechnic National University. He received the eng. degree in radio engineering from Lvov Polytechnic National University, Ukraine, in 1997. Currently he is on postdoctoral studying and works toward the Ph.D. degree. His research interests include digital image processing, radar imaging, antenna array, regularization of ill-posed problems, spectrum exstrapolation technique.
Lvov Polytechnic National University
S. Bandery st 12
Lvov, 79013, Ukraine